141 (2016) MATHEMATICA BOHEMICA No. 1, 91–98

AN ORDERED STRUCTURE OF PSEUDO-BCI-ALGEBRAS

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Received December 16, 2013
Communicated by Miroslav Ploščica

Abstract. In Chajda's paper (2014), to an arbitrary BCI-algebra the author assigned an ordered structure with one binary operation which possesses certain antitone mappings. In the present paper, we show that a similar construction can be done also for pseudo-BCI-algebras, but the resulting structure should have two binary operations and a set of couples of antitone mappings which are in a certain sense mutually inverse. The motivation for this approach is the well-known fact that every commutative BCK-algebra is in fact a join-semilattice and we try to obtain a similar result also for the non-commutative case and for pseudo-BCI-algebras which generalize BCK-algebras, see e.g. Imai and Iséki (1966) and Iséki (1966).

Keywords: pseudo-BCI-algebra; directoid; antitone mapping; pseudo-BCI-structure

MSC 2010: 06F35, 03G25

1. Introduction

The concept of a BCI-algebra was introduced by Iséki [9] in order to study implication fragments of non-classical logics. Pseudo-BCI-algebras were introduced by Dudek and Jun [5] as a reasonable generalization of BCI-algebras which enables an algebraic axiomatization of a larger class of logics including also fuzzy logics. Hence, the structure of BCI-algebras and of pseudo-BCI-algebras plays an important role in the study of these logics. The structure of BCI-algebras was already treated by the first author in [1]. However, as pointed out in [4], [6] and [7], also pseudo-BCI-algebras form an important tool for an algebraic axiomatization of implicational fragments of non-classical logics and hence we are motivated to reveal their structure. Moreover, the class of pseudo-BCI-algebras contains as a subclass

Support of the research by the Austrian Science Fund (FWF), project I 1923-N25, and the Czech Science Foundation (GAČR), project 15-34697L, is gratefully acknowledged.

DOI: 10.21136/MB.2016.7

the class of pseudo-BCK-algebras; thus we can follow the ideas of our paper [2]. In fact, we try an approach for going from BCI-algebras to pseudo-BCI-algebras similar to that used in [2] for going from BCK-algebras to pseudo-BCK-algebras, see [1] and [4].

Our goal is to convert every pseudo-BCI-algebra into a structure containing two binary operations each of them being similar to that of a directoid, see e.g. [3]. This was suggested by the fact that every commutative BCK-algebra is in fact a join-semilattice and directoids are the best approximation of semilattices in directed ordered sets where the existence of suprema is not necessarily assumed. Of course, our pseudo-BCI-algebras need not be commutative and are considerably weaker than BCK-algebras, thus one cannot expect that the corresponding structure will be a semilattice or a directoid.

Moreover, contrary to the case of pseudo-BCK-algebras, see [2], these binary oparations do not constitute common upper bounds of their operands. However, we were successful in finding a structure with two binary operations similar to that of a directoid and with a set of couples of unary operations, in fact antitone mappings, which are mutually inverse in a certain sense explained below. We show that there is a one-to-one correspondence between a pseudo-BCI-algebra and the derived structure in the sense that the given pseudo-BCI-algebra can be recovered from that structure.

2. Main results

We start with the definition of a pseudo-BCI-algebra.

Definition 2.1 (see e.g. [7]). A *pseudo*-BCI-*algebra* is an algebra $\mathcal{A} = (A, \to, \rightsquigarrow, 1)$ of type $(2, 2, 0)$ satisfying the following axioms:

(P1) $(x \to y) \rightsquigarrow ((y \to z) \rightsquigarrow (x \to z)) = 1$,
(P2) $(x \rightsquigarrow y) \to ((y \rightsquigarrow z) \to (x \rightsquigarrow z)) = 1$,
(P3) $1 \to x = x$,
(P4) $1 \rightsquigarrow x = x$,
(P5) $x \to y = y \to x = 1$ implies $x = y$.

We next show that $x \to y = 1$ if and only if $x \rightsquigarrow y = 1$.

Lemma 2.1. *Let $\mathcal{A} = (A, \to, \rightsquigarrow, 1)$ be a pseudo-BCI-algebra and $a, b \in A$. Then $a \to b = 1$ if and only if $a \rightsquigarrow b = 1$.*

Proof. $a \to b = 1$ implies $a \rightsquigarrow b = a \rightsquigarrow (1 \rightsquigarrow b) = (1 \to a) \rightsquigarrow ((a \to b) \rightsquigarrow (1 \to b)) = 1$ according to (P1), (P3) and (P4), and $a \rightsquigarrow b = 1$ implies $a \to b = a \to (1 \to b) = (1 \rightsquigarrow a) \to ((a \rightsquigarrow b) \to (1 \rightsquigarrow b)) = 1$ according to (P2), (P3) and (P4). □

R e m a r k 2.1. Lemma 2.1 implies that all the axioms of a pseudo-BCI-algebra are self-dual, i.e. the following axiom also holds:

(P5′) $x \rightsquigarrow y = y \rightsquigarrow x = 1$ implies $x = y$.

This is the reason for the following duality principle holding for these algebras:

Theorem 2.1 (Duality principle for pseudo-BCI-algebras)**.** *If an assertion holds for some expression in a pseudo-*BCI-*algebra* $\mathcal{A} = (A, \to, \rightsquigarrow, 1)$ *then the dual sentence obtained by interchanging* $\to$ *and* $\rightsquigarrow$ *holds, as well.*

In every pseudo-BCI-algebra one can define a partial order relation in a natural way.

Definition 2.2. Let $\mathcal{A} = (A, \to, \rightsquigarrow, 1)$ be a pseudo-BCI-algebra. Define a binary relation $\leqslant$ on A by $x \leqslant y$ if and only if $x \to y = 1$, $x, y \in A$.

Lemma 2.2. *Let* $\mathcal{A} = (A, \to, \rightsquigarrow, 1)$ *be a pseudo-*BCI-*algebra. Then* $(A, \leqslant)$ *is a poset.*

P r o o f. Let $a, b, c \in A$. Then $a \to a = 1 \to (a \to a) = (1 \rightsquigarrow 1) \to ((1 \rightsquigarrow a) \to (1 \rightsquigarrow a)) = 1$ according to (P2), (P3) and (P4) and hence $\leqslant$ is reflexive. Because of (P5), $\leqslant$ is antisymmetric. If $a \leqslant b \leqslant c$ then $a \to b = b \to c = 1$ and hence $a \to c = 1 \rightsquigarrow (a \to c) = 1 \rightsquigarrow (1 \rightsquigarrow (a \to c)) = (a \to b) \rightsquigarrow ((b \to c) \rightsquigarrow (a \to c)) = 1$ according to (P1) and (P4), which implies $a \leqslant c$ showing transitivity of $\leqslant$. □

Due to Theorem 2.1 we have also $x \leqslant y$ if and only if $x \rightsquigarrow y = 1$, $x, y \in A$.

Next we define two binary operations on any pseudo-BCI-algebra.

Definition 2.3. Let $\mathcal{A} = (A, \to, \rightsquigarrow, 1)$ be a pseudo-BCI-algebra. Define binary operations $\sqcup$ and $\cup$ on A by $x \sqcup y := (x \to y) \rightsquigarrow y$ and $x \cup y := (x \rightsquigarrow y) \to y$, $x, y \in A$.

That these two operations need not coincide can be seen from the following

E x a m p l e 2.1. On the four-element set $A := \{0, a, b, 1\}$ define two binary operations $\to$ and $\rightsquigarrow$ as follows:

$\to$	0	a	b	1
0	1	1	1	1
a	b	1	b	1
b	0	a	1	1
1	0	a	b	1

and

$\rightsquigarrow$	0	a	b	1
0	1	1	1	1
a	0	1	b	1
b	a	a	1	1
1	0	a	b	1

It can be easily checked that $\mathcal{A} = (A, \to, \rightsquigarrow, 1)$ is a pseudo-BCI-algebra, but

$$a \sqcup 0 = (a \to 0) \rightsquigarrow 0 = b \rightsquigarrow 0 = a \neq 1 = 0 \to 0 = (a \rightsquigarrow 0) \to 0 = a \cup 0.$$

Now we list some properties of pseudo-BCI-algebras.

Lemma 2.3. *Let $\mathcal{A} = (A, \to, \rightsquigarrow, 1)$ be a pseudo-*BCI*-algebra and $a, b, c \in A$. Then the following assertions (and their dual statements obtained by interchanging $\to$ and $\rightsquigarrow$ as well as $\sqcup$ and $\cup$) hold:*

(i) $a \leqslant a \sqcup b$,
(ii) $a \leqslant b$ *if and only if* $a \sqcup b = b$,
(iii) $a \leqslant b$ *implies* $b \to c \leqslant a \to c$ *and* $c \to a \leqslant c \to b$,
(iv) $a \leqslant b$ *implies* $a \sqcup c \leqslant b \sqcup c$,
(v) $((a \to b) \rightsquigarrow b) \to b = a \to b$,
(vi) $a \to (b \rightsquigarrow c) = b \rightsquigarrow (a \to c)$.

Proof. Properties (iii), (v) and (vi) are proved in [5], Proposition 3.2.

(i) $a \rightsquigarrow (a \sqcup b) = a \rightsquigarrow ((a \to b) \rightsquigarrow b) = (1 \to a) \rightsquigarrow ((a \to b) \rightsquigarrow (1 \to b)) = 1$ according to (P1) and (P3).

(ii) If $a \leqslant b$ then $a \to b = 1$ and hence $a \sqcup b = (a \to b) \rightsquigarrow b = 1 \rightsquigarrow b = b$ according to (P4). If, conversely, $a \sqcup b = b$ then $a \leqslant a \sqcup b = b$ according to (i).

(iv) $a \leqslant b$ implies $b \to c \leqslant a \to c$ according to (iii) and hence $a \sqcup c = (a \to c) \rightsquigarrow c \leqslant (b \to c) \rightsquigarrow c = b \sqcup c$ according to (iii) and Theorem 2.1. □

Next we list some properties of $\sqcup$. We remark that the dual statements obtained by replacing $\sqcup$ by $\cup$ hold, as well.

Lemma 2.4. *Let $\mathcal{A} = (A, \to, \rightsquigarrow, 1)$ be a pseudo-*BCI*-algebra and $a, b, c \in A$. Then the following assertions hold:*

(i) $a \sqcup a = a$,
(ii) $a \sqcup b = b$ *and* $b \sqcup a = a$ *together imply* $a = b$,
(iii) $(a \sqcup b) \sqcup b = a \sqcup (a \sqcup b) = a \sqcup b$,
(iv) $(a \sqcup c) \sqcup ((a \sqcup b) \sqcup c) = (a \sqcup b) \sqcup c$,
(v) $1 \sqcup a = 1$.

Proof. (i) $a \sqcup a = (a \to a) \rightsquigarrow a = 1 \rightsquigarrow a = a$ according to Lemma 2.2 and (P4).

(ii) Follows from Lemma 2.2 and from (ii) of Lemma 2.3.

(iii) According to (v) of Lemma 2.3 we have $(a \sqcup b) \sqcup b = (((a \to b) \rightsquigarrow b) \to b) \rightsquigarrow b = (a \to b) \rightsquigarrow b = a \sqcup b$. The rest follows from (i) and (ii) of Lemma 2.3.

(iv) We have $a \leqslant a \sqcup b$ according to (i) of Lemma 2.3. This implies $a \sqcup c \leqslant (a \sqcup b) \sqcup c$ according to (iv) of Lemma 2.3. The rest follows from (ii) of Lemma 2.3.

(v) $1 \sqcup a = (1 \to a) \rightsquigarrow a = a \rightsquigarrow a = 1$ according to (P3), Lemma 2.2 and Theorem 2.1. □

R e m a r k 2.2. Let us note that for a pseudo-BCI-algebra $\mathcal{A} = (A, \to, \rightsquigarrow, 1)$ the derived structure $(A, \sqcup)$ is not a directoid in general because it need not satisfy the identity $y \sqcup (x \sqcup y) = x \sqcup y$, see [3] for details. Moreover, 1 need not be the greatest element in the derived ordered set $(A, \leqslant)$ since $x \sqcup 1$ need not be equal to 1. In fact, this is just the case when $\mathcal{A}$ is a pseudo-BCK-algebra.

E x a m p l e 2.2 (cf. [6]). Define binary operations $\to$ and $\rightsquigarrow$ on $\mathbb{R}^2$ by

$$(x, y) \to (z, u) := (z - x, (u - y)\mathrm{e}^{-x}) \quad \text{and} \quad (x, y) \rightsquigarrow (z, u) := (z - x, u - y\mathrm{e}^{z-x})$$

$((x, y), (z, u) \in \mathbb{R}^2)$. Then it can be easily checked that $\mathcal{A} := (\mathbb{R}^2, \to, \rightsquigarrow, (0, 0))$ is a pseudo-BCI-algebra which is obviously not a BCI-algebra. Let $(a, b), (c, d) \in \mathbb{R}^2$. The algebra $\mathcal{A}$ is not a pseudo-BCK-algebra since

$$(a, b) \to (0, 0) = (-a, (-b)\mathrm{e}^{-a}) \neq (0, 0)$$

in case $(a, b) \neq (0, 0)$. Moreover, it can be easily checked that $(a, b) \sqcup (c, d) = (a, b)$. This shows

$$(a, b) \sqcup ((c, d) \sqcup (a, b)) = (a, b) \neq (c, d) = (c, d) \sqcup (a, b)$$

in case $(a, b) \neq (c, d)$.

On each pseudo-BCI-algebra we define two unary operations as follows:

Definition 2.4. Let $\mathcal{A} = (A, \to, \rightsquigarrow, 1)$ be a pseudo-BCI-algebra. For every $x \in A$ define unary operations f_x and g_x on A by $f_x(y) := y \to x$ and $g_x(y) := y \rightsquigarrow x$ for all $y \in A$.

R e m a r k 2.3. Because of (iii) of Lemma 2.3 and Theorem 2.1, f_x and g_x are antitone. Moreover, $g_x(f_x(y \sqcup x)) = y \sqcup x$ and $f_x(g_x(y \cup x)) = y \cup x$ for all $x, y \in A$ according to (v) of Lemma 2.3. If $x \leqslant y$ then, by (i) of Lemma 2.4 and (iv) of Lemma 2.3, we have $x = x \sqcup x \leqslant y \sqcup x$. Hence, f_x and g_x are mutually inverse with respect to those elements of $[x) := \{z \in A;\ x \leqslant z\}$ which are of the form $y \sqcup x$ or $y \cup x$.

We list some properties of the unary operations just defined.

Lemma 2.5. *Let $\mathcal{A} = (A, \to, \rightsquigarrow, 1)$ be a pseudo-*BCI*-algebra and $a, b, c \in A$. Then the following assertions (and their dual statements obtained by interchanging $\to$ and $\rightsquigarrow$ as well as $\sqcup$ and $\cup$ as well as f_x and g_x) hold:*

(i) $f_a(a) = 1$ *and* $f_a(1) = a$,

(ii) $f_b(a \sqcup b) = a \to b$,

(iii) $g_b(f_b(a \sqcup b) \cup b) = g_b(f_b(a \sqcup b)) = a \sqcup b$,

(iv) $f_b(a \sqcup b) \cup f_{b \sqcup c}((a \sqcup c) \sqcup (b \sqcup c)) = f_{b \sqcup c}((a \sqcup c) \sqcup (b \sqcup c))$,

(v) $f_{g_c(b \cup c)}(a \sqcup g_c(b \cup c)) = g_{f_c(a \sqcup c)}(b \cup f_c(a \sqcup c))$,

(vi) $f_b((a \sqcup b) \sqcup b) = f_b(a \sqcup b)$,

(vii) $f_a((a \sqcup b) \sqcup a) = f_a(a \sqcup b)$.

Proof. (i) $f_a(a) = a \to a = 1$ according to Lemma 2.2 and $f_a(1) = 1 \to a = a$ according to (P3).

(ii) $f_b(a \sqcup b) = ((a \to b) \rightsquigarrow b) \to b = a \to b$ according to (v) of Lemma 2.3.

(iii) $g_b(f_b(a \sqcup b) \cup b) = g_b(f_b(a \sqcup b)) = (a \to b) \rightsquigarrow b = a \sqcup b$ according to (v) of Lemma 2.3 and (ii) of Lemma 2.5.

(iv) $f_{b \sqcup c}((a \sqcup c) \sqcup (b \sqcup c)) = (a \sqcup c) \to (b \sqcup c) = ((a \to c) \rightsquigarrow c) \to ((b \to c) \rightsquigarrow c) = (b \to c) \rightsquigarrow (((a \to c) \rightsquigarrow c) \to c) = (b \to c) \rightsquigarrow (a \to c)$ according to (v) and (vi) of Lemma 2.3 and (ii) of Lemma 2.5. Now (iv) follows from (ii) of Lemma 2.3, (P1), (ii) of Lemma 2.5 and from Theorem 2.1.

(v) $f_{g_c(b \cup c)}(a \sqcup g_c(b \cup c)) = a \to (b \rightsquigarrow c) = b \rightsquigarrow (a \to c) = g_{f_c(a \sqcup c)}(b \cup f_c(a \sqcup c))$ according to (vi) of Lemma 2.3, Theorem 2.1 and (ii) of Lemma 2.5.

(vi) $f_b((a \sqcup b) \sqcup b) = (a \sqcup b) \to b = f_b(a \sqcup b)$ according to (ii).

(vii) $f_a((a \sqcup b) \sqcup a) = (a \sqcup b) \to a = f_a(a \sqcup b)$ according to (ii). $\square$

Now we define the notion of a pseudo-BCI-structure which is similar to a semilattice equipped with antitone mutually inverse mappings.

Definition 2.5. A *pseudo*-BCI-*structure* is an ordered sixtuple $(A, \sqcup, \cup, (f_x; x \in A), (g_x; x \in A), 1)$ where $(A, \sqcup, \cup, 1)$ is an algebra of type $(2, 2, 0)$ and for any $x \in A$, f_x and g_x are unary operations on A such that the following axioms (and their dual formulations obtained by interchanging $\sqcup$ and $\cup$ as well as f_x and g_x) are satisfied:

(S1) $x \sqcup y = y$ and $y \sqcup x = x$ together imply $x = y$,

(S2) $1 \sqcup x = 1$,

(S3) $f_x(x) = 1$ and $f_x(1) = x$,

(S4) $g_y(f_y(x \sqcup y) \cup y) = g_y(f_y(x \sqcup y)) = x \sqcup y$,

(S5) $f_y(x \sqcup y) \cup f_{y \sqcup z}((x \sqcup z) \sqcup (y \sqcup z)) = f_{y \sqcup z}((x \sqcup z) \sqcup (y \sqcup z))$,

(S6) $f_{g_z(y \cup z)}(x \sqcup g_z(y \cup z)) = g_{f_z(x \sqcup z)}(y \cup f_z(x \sqcup z))$,

(S7) $f_y((x \sqcup y) \sqcup y) = f_y(x \sqcup y)$.

To every pseudo-BCI-algebra we can assign a pseudo-BCI-structure.

Theorem 2.2. *Let* $\mathcal{A} = (A, \to, \rightsquigarrow, 1)$ *be a pseudo-*BCI*-algebra. Then* $\mathbf{S}(\mathcal{A}) := (A, \sqcup, \cup, (f_x; x \in A), (g_x; x \in A), 1)$ *is a pseudo-*BCI*-structure.*

P r o o f. Axioms (S1) and (S2) follow from Lemma 2.4 and (S3)–(S7) from Lemma 2.5. □

Conversely, to every pseudo-BCI-structure we can assign a pseudo-BCI-algebra.

Theorem 2.3. *Let* $\mathcal{S} := (S, \sqcup, \cup, (f_x; x \in S), (g_x; x \in S), 1)$ *be a pseudo-*BCI*-structure. Define*

$$x \to y := f_y(x \sqcup y) \quad \text{and} \quad x \rightsquigarrow y := g_y(x \cup y)$$

for all $x, y \in S$. *Then* $\mathbf{A}(\mathcal{S}) := (S, \to, \rightsquigarrow, 1)$ *is a pseudo-*BCI*-algebra.*

P r o o f. Let $a, b, c \in S$. If $a \to b = 1$ then $f_b(a \sqcup b) = 1$ and hence $a \sqcup b = g_b(f_b(a \sqcup b)) = g_b(1) = b$ according to (S3) and (S4). If, conversely, $a \sqcup b = b$ then $a \to b = f_b(a \sqcup b) = f_b(b) = 1$ according to (S3). Hence $a \to b = 1$ if and only if $a \sqcup b = b$.

(P1) We have $(a \to b) \rightsquigarrow b = g_b(f_b(a \sqcup b) \cup b) = a \sqcup b$ according to (S4),
$((a \to b) \rightsquigarrow b) \to b = (a \sqcup b) \to b = f_b((a \sqcup b) \sqcup b) = f_b(a \sqcup b) = a \to b$ according to (S7) and
$a \to (b \rightsquigarrow c) = f_{g_c(b \cup c)}(a \sqcup g_c(b \cup c)) = g_{f_c(a \sqcup c)}(b \cup f_c(a \sqcup c)) = b \rightsquigarrow (a \to c)$ according to (S6).
Now $(a \sqcup c) \to (b \sqcup c) = ((a \to c) \rightsquigarrow c) \to ((b \to c) \rightsquigarrow c) = (b \to c) \rightsquigarrow (((a \to c) \rightsquigarrow c) \to c) = (b \to c) \rightsquigarrow (a \to c)$.
From (S5) we conclude $(a \to b) \cup ((a \sqcup c) \to (b \sqcup c)) = (a \sqcup c) \to (b \sqcup c)$ which implies $(a \to b) \cup ((b \to c) \rightsquigarrow (a \to c)) = (b \to c) \rightsquigarrow (a \to c)$, i.e., (P1) follows by Theorem 2.1.

(P2) Follows by duality.

(P3) $1 \to a = f_a(1 \sqcup a) = f_a(1) = a$ according to (S2) and (S3).

(P4) Follows by duality.

(P5) If $a \to b = b \to a = 1$ then $a \sqcup b = b$ and $b \sqcup a = a$ and hence $a = b$ according to (S1). □

Finally, we show that if we start with a pseudo-BCI-algebra, construct its corresponding pseudo-BCI-structure and then assign to this its corresponding pseudo-BCI-algebra then we obtain the original one.

Theorem 2.4. *Let $\mathcal{A}=(A,\to,\rightsquigarrow,1)$ be a pseudo-*BCI*-algebra. Then $\mathbf{A}(\mathbf{S}(\mathcal{A}))=\mathcal{A}$.*

Proof. Put $\mathbf{S}(\mathcal{A}) = (A, \sqcup, \cup, (f_x; x \in A), (g_x; x \in A), 1)$ and $\mathbf{A}(\mathbf{S}(\mathcal{A})) = (A, \to', \rightsquigarrow', 1)$ and let $a, b \in A$. Then

$$a \to' b = f_b(a \sqcup b) = ((a \to b) \rightsquigarrow b) \to b = a \to b$$

according to (v) of Lemma 2.3. The equality $a \rightsquigarrow' b = a \rightsquigarrow b$ follows by duality. $\square$

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