FACTORIZATIONS OF NORMALITY VIA GENERALIZATIONS OF β-NORMALITY

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Abstract. The notion of β-normality was introduced and studied by Arhangel'skii, Ludwig in 2001. Recently, almost β-normal spaces, which is a simultaneous generalization of β-normal and almost normal spaces, were introduced by Das, Bhat and Tartir. We introduce a new generalization of normality, namely weak β-normality, in terms of θ-closed sets, which turns out to be a simultaneous generalization of β-normality and θ-normality. A space X is said to be weakly β-normal (wβ-normal) if for every pair of disjoint closed sets A and B out of which, one is θ-closed, there exist open sets U and V such that $\overline{A \cap U} = A$, $\overline{B \cap V} = B$ and $\overline{U} \cap \overline{V} = \emptyset$. It is shown that w$\beta$-normality acts as a tool to provide factorizations of normality.

Keywords: normal space; (weakly) densely normal space; (weakly) θ-normal space; almost normal space; almost β-normal space; κ-normal space; (weakly) β-normal space

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1. Introduction and Preliminaries

Factorization of a given topological property in terms of two weaker properties is a technique to investigate properly the existing notions of general topology. In [14], Singal and Arya have given a factorization of normality in terms of almost normality and semi-normality. Kohli and Das in [8] have introduced some variants of θ-normality which were utilized in the paper to factorize normality. δ-normal spaces were introduced by Mack in [12], and it was utilized by Kohli and Singh in [11] to provide one more factorization of normality. In [4], Das studied spaces between normality and κ-normality and obtained some decompositions of normality.

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A generalization of normality, namely β-normality, was introduced by Arhangel'skii, Ludwig in [2], and a factorization of normal spaces was obtained in terms of κ-normal spaces. Recently, a simultaneous generalization of β-normality and almost normality, namely almost β-normality, which was instrumental to obtain another factorization of normality, was introduced in [6]. In this paper, we introduce the notion of weak β-normality which is a simultaneous generalization of β-normality and θ-normality. Further, weak β-normality and almost β-normality also serve as necessary ingredients towards the factorization of normality in terms of relative topological properties such as densely normal spaces (see [1]) and weakly densely normal spaces (see [5]) which lies in between normality and κ-normality.

Let X be a topological space and let A be a subset of X. Throughout the present paper, the closure of a set A will be denoted by $\overline{A}$ or cl$A$, and the interior of a set A will be denoted by int$A$ or A°. A point $x \in X$ is said to be a θ-limit point of A (see [18]) if every closed neighbourhood of X intersects A. The θ-closure of A is denoted by $\mathrm{cl}_\theta A$. A set A is said to be θ-closed if $A = \mathrm{cl}_\theta A$. The complement of a θ-closed set is known as θ-open set. A set A is said to be regularly closed if $\overline{A^\circ} = A$. The complement of a regularly closed set is called regularly open set. Let A and Y be subsets of X. Then A is said to be concentrated on Y (see [1]) if $A \subset \overline{A \cap Y}$. A subset A of Y is said to be strongly concentrated on Y (see [5]) if $A \subset \overline{(A \cap Y)^\circ}$. It is obvious that every strongly concentrated set is concentrated. A space X is normal on Y (see [1]) if every two disjoint closed subsets of X concentrated on Y can be separated by disjoint open neighbourhoods in X. Similarly, X is said to be weakly normal on Y (see [5]) if for every disjoint closed subsets A and B of X strongly concentrated on Y, there exist disjoint open sets in X separating A and B, respectively.

Lemma 1.1 ([7]). *A topological space X is regular if and only if every closed set in X is θ-closed.*

Definition 1.2. A topological space X is said to be

(i) θ-normal (see [8]) if every pair of disjoint closed sets, one of which is θ-closed, are contained in disjoint open sets,

(ii) weakly θ-normal (wθ-normal) (see [8]) if every pair of disjoint θ-closed sets is contained in disjoint open sets,

(iii) κ-normal (see [16]) (mildly normal, see [15]) if any two disjoint regularly closed sets in X have disjoint open neighbourhoods.

Definition 1.3. A topological space X is said to be

(i) β-normal (see [2]) if for any two disjoint closed subsets A and B of X, there exist open subsets U and V of X such that $A \cap U$ is dense in A, $B \cap V$ is dense in B and $\overline{U} \cap \overline{V} = \emptyset$,

(ii) almost β-normal (see [6]) if for every pair of disjoint closed sets A and B, one of which is regularly closed, there exist open sets U and V such that $\overline{A \cap U} = A$, $\overline{B \cap V} = B$ and $\overline{U} \cap \overline{V} = \emptyset$.

Definition 1.4. A topological space X is said to be

(i) densely normal (see [1]) if there is a dense subspace Y in X such that X is normal on Y,

(ii) weakly densely normal (see [5]) if there exists a proper dense subspace Y of X such that X is weakly normal on Y.

Although, densely normal and weakly densely normal spaces are relative topological properties, in [5] it is observed that every densely normal space is weakly densely normal and every weakly densely normal space is κ-normal.

2. Weak β-normality

Definition 2.1. A space X is said to be weakly β-normal (wβ-normal) if for every pair of disjoint closed sets A and B, out of which one is θ-closed, there exist open sets U and V such that $\overline{A \cap U} = A$, $\overline{B \cap V} = B$ and $\overline{U} \cap \overline{V} = \emptyset$.

Theorem 2.2. *Every θ-normal space is wβ-normal.*

Proof. Let X be a θ-normal space and let A and B be two disjoint closed sets, out of which B is θ-closed. Since $X - B$ is a θ-open set containing the closed set A and X is θ-normal, by Theorem 3.3 of [8], there exists an open set U such that $A \subset U \subset \overline{U} \subset X - B$. Again, by applying θ-normality of X, there exists an open set V such that $\overline{U} \subset V \subset \overline{V} \subset X - B$. Let $W = X - \overline{V}$. Thus $B \subset W \subset \overline{W} \subset X - V$. Since $\overline{U} \cap X - V = \emptyset$, U and V are two disjoint open sets such that $\overline{A \cap U} = A$, $\overline{B \cap W} = B$ and $\overline{U} \cap \overline{W} = \emptyset$. Hence, X is wβ-normal. □

The above result and the following example establishes that every θ-normal space is wβ-normal but need not be β-normal. Also, there exist β-normal spaces which are not θ-normal as discussed below in Example 2.6. Thus, θ-normality and β-normality are independent notions.

Example 2.3. Let us consider the modified Fort space (see [17]) in which $X = \mathbb{Y} \cup \{p, q\}$, where $\mathbb{Y}$ is an infinite set and p, q are two distinct points. Topologize X by taking any open subset of $\mathbb{Y}$ and any open subset containing p or q if and only if it contains all but a finite number of points in $\mathbb{Y}$. Here, the only θ-closed sets are either finite in $\mathbb{Y}$ or of the form $\{p, q\} \cup D$, where the complement of D is finite in $\mathbb{Y}$. The space is θ-normal, as a θ-closed set disjoint from a closed set can be separated by disjoint open sets. But it is not β-normal, as for two disjoint closed sets $A = \{p\} \cup E$, $B = \{q\} \cup F$, where E and F are finite subsets of $\mathbb{Y}$, there do not exist two open sets satisfying the required conditions of β-normality.

Theorem 2.4. *Every β-normal space is wβ-normal.*

Proof. Since every θ-closed set is closed, the proof is obvious. $\square$

The following interrelation among the variants of the normality is obvious from the definitions and implications shown above. But none of these implications is reversible (see [6], [5], [8], [14] and Example 2.5, Example 2.6 below).

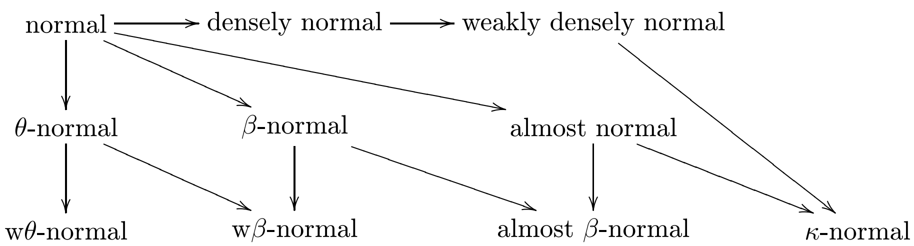

Figure 1. Interrelation diagram

Example 2.5. A wβ-normal space which is not β-normal. Let X be the set of integers. Define a topology τ on X, where every odd integer is open and a set U is open if for every even integer $p \in U$, the successor and the predecessor of p also belongs to U. Since there does not exist a θ-closed set disjoint from a closed set, the space is vacuously wβ-normal. But X is not β-normal, as for disjoint closed sets $A = \{2\}$, $B = \{4\}$ there do not exist disjoint open sets intersecting A and B, respectively.

Example 2.6. A wβ-normal space which is not θ-normal. Consider the space discussed in Example 1 of [13], in which the space is β-normal Tychonoff space but not normal. Since the space is β-normal, it is wβ-normal. But it cannot be θ-normal, as the space is a Tychonoff non-normal space and it is known from [8] that every regular θ-normal space is normal.

The notions of wθ-normality and wβ-normality are independent of each other which is evident from Example 2.7 and Remark 2.8. But in the class of wθ-regular spaces, every wθ-normal space is wβ-normal.

Example 2.7. A wθ-normal space which is not wβ-normal. Let X denote the interior S° of the unit square S in the plane together with the points $(0,0)$ and $(1,0)$, i.e. $X = S^\circ \cup \{(0,0),(1,0)\}$. Every point in S° has the usual Euclidean neighbourhoods. The points $(0,0)$ and $(1,0)$ have neighborhoods of the form U_n and V_n, respectively, where

$$U_n = \{(0,0)\} \cup \Big\{(x,y)\colon\ 0 < x < \frac{1}{2},\ 0 < y < \frac{1}{n}\Big\},$$
$$V_n = \{(1,0)\} \cup \Big\{(x,y)\colon\ \frac{1}{2} < x < 1,\ 0 < y < \frac{1}{n}\Big\}.$$

The space is wθ-normal (see [8]), but not wβ-normal because for the θ-closed sets, $A = (0,0)$ disjoint from the closed set $B = (1,0)$, we cannot find disjoint open sets for A and B satisfying the conditions of β-normality.

Remark 2.8. The space discussed in Example 2.6 is wβ-normal and fails to be wθ-normal because every regular wθ-normal space is normal (see [8]) and the space discussed in Example 2.6 is regular. So the notions of wθ-normality and wβ-normality are independent notions.

Remark 2.9. The notions of wβ-normality and almost β-normality are independent of each other as the Example 2.5 is wβ-normal but not almost β-normal because for closed set $A = \{6\}$ and regularly closed set $B = \{2,3,4\}$, there do not exist disjoint open sets U and V satisfying $\overline{A \cap U} = A$, $\overline{B \cap V} = B$, and $\overline{U} \cap \overline{V} = \emptyset$. The space discussed in Example 2.7 is not wβ-normal but is almost normal. So it is almost β-normal.

The following notion of θ-regularity was introduced in [8]. It is a simultaneous generalization of regularity as well as normality and it was further studied and few more generalizations were given in [3]. These variants of θ-regularities are useful in the sequel to factorize normality.

Definition 2.10 ([8]). A topological space X is said to be θ-regular if for each closed set F and each open set U containing F, there exists a θ-open set V such that $F \subset V \subset U$.

Definition 2.11 ([9]). A topological space X is said to be weakly θ-regular (wθ-regular) if for each θ-closed set F and for each open set U containing F, there exists a θ-open set V such that $F \subset V \subset U$.

Theorem 2.12 ([8]). *A topological space X is wθ-normal if and only if for every θ-closed set A and a θ-open set U containing A, there is an open set V such that $A \subset V \subset \overline{V} \subset U$.*

Theorem 2.13. *A wθ-regular, wθ-normal space is wβ-normal.*

P r o o f. Let X be wθ-regular, wθ-normal space and let A, B be two disjoint closed sets, out of which B is θ-closed. By wθ-regularity, there exists a θ-open set E such that $B \subset E \subset X - A$. By w$\theta$-normality of X, for θ-open set E containing B, there exists an open set P such that $B \subset P \subset \overline{P} \subset E$. Since $B \subset P \subset \overline{P} \subset E \subset X - A$, $A \subset X - E \subset X - \overline{P}$. Thus, $X - \overline{P}$ is an open set containing the θ-closed set $X - E$. Thus, by wθ-regularity of X, there exists a θ-open set F such that $A \subset X - E \subset F \subset X - \overline{P}$. Again, applying w$\theta$-normality of X, there exists an open set Q such that $A \subset X - E \subset Q \subset \overline{Q} \subset F \subset X - \overline{P}$. Here, P and Q are two disjoint open sets in X such that $\overline{A \cap Q} = A$, $\overline{B \cap P} = B$ and $\overline{P} \cap \overline{Q} = \emptyset$. Hence, X is wβ-normal. □

Corollary 2.14. *An almost compact, wθ-regular space is wβ-normal.*

P r o o f. Every almost compact space is wθ-normal (see [8]). The proof follows from Theorem 2.13. □

Recall that a space is extremely disconnected if the closure of every open set is open.

Theorem 2.15. *In an extremely disconnected space, every wβ-normal space is θ-normal.*

P r o o f. Let X be an extremely disconnected wβ-normal space and let A, B be two disjoint closed sets, out of which B is θ-closed. Since the space is wβ-normal, there exist two disjoint open sets U and V such that $\overline{A \cap U} = A$, $\overline{B \cap V} = B$, and $\overline{U} \cap \overline{V} = \emptyset$. Thus, by extremely disconnectedness of X, $\overline{U}$ and $\overline{V}$ are two disjoint open sets containing A and B, respectively. Hence, the space is θ-normal. □

Corollary 2.16. *In an extremely disconnected space, every β-normal space is θ-normal.*

Theorem 2.17. *Every regular, wβ-normal space is β-normal.*

P r o o f. Since every closed set is θ-closed in a regular space (see [7]), the proof is obvious. □

Theorem 2.18. *In the class of almost regular spaces, a space is θ-normal if and only if it is wθ-normal and wβ-normal.*

P r o o f. If X is θ-normal, then obviously it is wθ-normal and wβ-normal.

Conversely, let X be an almost regular space which is wθ-normal as well as wβ-normal. Let A and B be two disjoint closed subsets of X out of which B is θ-closed.

By wβ-normality of X, there exist disjoint open sets U and V such that $\overline{U \cap A} = A$, $\overline{V \cap B} = B$ and $\overline{U} \cap \overline{V} = \emptyset$. Clearly, $A \subset \overline{U}$ and $B \subset \overline{V}$. Since X is almost regular, $\overline{U}$ is θ-closed by Theorem 2.5 of [10]. Thus, by wθ-normality of X, two disjoint θ-closed sets B and $\overline{U}$ can be separated by disjoint open sets. Hence, X is θ-normal. □

Theorem 2.19. *In the class of almost regular spaces, the following statements are equivalent:*

(i) *X is wβ-normal.*

(ii) *For each closed set A and for all θ-open subset U of X with $A \subseteq U$, there exists an open set V such that $\overline{V \cap A} = A \subseteq \overline{V} \subseteq U$.*

Proof. To prove (i) ⇒ (ii), suppose X is wβ-normal space. Let A be a closed subset of X contained in a θ-open subset U of X. Put $B = X - U$. Since B is θ-closed and X is wβ-normal, there exist open sets P and Q such that $\overline{P \cap A} = A$, $\overline{Q \cap B} = B$, $\overline{P} \cap \overline{Q} = \emptyset$. Thus, $\overline{P} \cap B = \emptyset$, which implies $\overline{P} \subseteq U$.

For (ii) ⇒ (i), let A and B be two closed subsets of X, out of which A is θ-closed. Since $X - A$ is θ-open and $B \subseteq X - A$, by the hypothesis, there exists an open set P such that $\overline{P \cap B} = B \subset \overline{P} \subset X - A$. $X - \overline{P}$ is open in X and contains A. By almost regularity of X, $X - \overline{P}$ is θ-open. So by the hypothesis, there exists an open set Q such that $\overline{A \cap Q} = A \subset \overline{Q} \subset X - \overline{P}$. Since $\overline{P} \cap \overline{Q} = \emptyset$, the space is w$\beta$-normal. □

Theorem 2.20. *In the class of wθ-regular spaces, the following statements are equivalent:*

(i) *X is wβ-normal.*

(ii) *For each closed set A and for all θ-open subset U of X with $A \subseteq U$, there exists an open set V such that $\overline{V \cap A} = A \subseteq \overline{V} \subseteq U$.*

Proof. To prove (i) ⇒ (ii), suppose X is wβ-normal space. Let A be a closed subset of X and U a θ-open subset of X such that $A \subseteq U$. Put $B = X - U$. Since B is θ-closed and X is wβ-normal, there exist open sets P and Q in X with disjoint closures such that $\overline{P \cap A} = A$, $\overline{Q \cap B} = B$. Thus, $\overline{P} \cap B = \emptyset$, which implies $\overline{P} \subseteq U$.

For (ii) ⇒ (i), let A and B be two disjoint closed subsets of X, out of which A is θ-closed. Since $X - A$ is θ-open and $B \subseteq X - A$, by the hypothesis, there exists an open set P such that $\overline{P} \subseteq X - A$ and $\overline{P \cap B} = B$. $X - \overline{P}$ is open in X and contains the θ-closed set A. By wθ-regularity of X, there exists a θ-open set V such that $A \subset V \subset X - \overline{P}$. By the hypothesis, there exists an open set Q such that $\overline{Q \cap A} = A$ and $\overline{Q} \subset V$. Since $\overline{P} \cap \overline{Q} = \emptyset$, the space is w$\beta$-normal. □

Theorem 2.21. *A wβ-normal, κ-normal space is θ-normal.*

P r o o f. Let A and B be two disjoint closed sets, out of which A is θ-closed. By wβ-normality of X, there exist disjoint open sets such that $\overline{U \cap A} = A$, $\overline{V \cap B} = B$ and $\overline{U} \cap \overline{V} = \emptyset$. This implies $A \subset \overline{U}$ and $B \subset \overline{V}$. Thus $\overline{U}$ and $\overline{V}$ are two disjoint regularly closed sets in X. By κ-normality of X, there exist disjoint open sets P and Q such that $\overline{U} \subset P$ and $\overline{V} \subset Q$. Therefore P and Q are disjoint open sets separating A and B, respectively. Hence, X is θ-normal. □

3. Factorizations of normality

Following Theorems provide factorizations of normality in terms of variants of normality such as wβ-normality and almost β-normality in presence or absence of other separation axioms.

Theorem 3.1. *In the class of Hausdorff, κ-normal spaces, the following statements are equivalent:*
(i) *X is normal.*
(ii) *X is θ-normal.*
(iii) *X is β-normal.*
(iv) *X is wβ-normal.*

P r o o f. (i) $\Rightarrow$ (ii) $\Rightarrow$ (iv) and (i) $\Rightarrow$ (iii) $\Rightarrow$ (iv) are obvious. To prove (iv) $\Rightarrow$ (ii), let X be a κ-normal, wβ-normal. Then by Theorem 2.21, X is θ-normal. The prove of (ii) $\Rightarrow$ (i) immediately follows from the result that in a T_2 space, every θ-normal space is normal (see [8]). □

Theorem 3.2. *In the class of Hausdorff almost regular spaces, a space is normal if and only if it is wθ-normal and wβ-normal.*

P r o o f. The proof immediately follows from Theorem 2.18 and Theorem 3.5 from [8] and the fact that in a Hausdorff space, θ-normality coincide with normality. □

Theorem 3.3. *A space is normal if and only if it is wβ-normal, κ-normal and θ-regular.*

P r o o f. Let X be a normal space. Then it is wβ-normal, κ-normal and θ-regular. Conversely, let X be a wβ-normal, κ-normal and θ-regular space. Let A and B be two disjoint closed sets of X. Thus, A is closed set contained in an open set $X - B$. By θ-regularity of X, there exists a θ-open set W such that $A \subset W \subset X - B$. Here, $D = X - W$ is a θ-closed set containing B and $D \cap A = \emptyset$. By wβ-normality of X, there exist two disjoint open sets U and V such that $\overline{D \cap U} = D$, $\overline{A \cap V} = A$ and $\overline{U} \cap \overline{V} = \emptyset$. Here, $D \subset \overline{U}$ and $A \subset \overline{V}$. Since closure of an open set is regularly closed, $\overline{U}$ and $\overline{V}$ are two disjoint regularly closed sets of X. By κ-normality of X, there exist disjoint open sets P and Q separating $\overline{U}$ and $\overline{V}$, respectively. As $B \subset D \subset \overline{U}$ and $A \subset \overline{V}$, the space is normal. □

Corollary 3.4. *In the class of θ-regular, wβ-normal spaces, the following statements are equivalent:*
(i) *X is normal.*
(ii) *X is densely normal.*
(iii) *X is weakly densely normal.*
(iv) *X is almost normal.*
(v) *X is κ-normal.*

Proof. The implications (i) $\Rightarrow$ (ii) $\Rightarrow$ (iii) $\Rightarrow$ (v) and (i) $\Rightarrow$ (iv) $\Rightarrow$ (v) are obvious from [5] and [15], respectively. To prove (v) $\Rightarrow$ (i), let X be a θ-regular, wβ-normal and κ-normal space. Thus, by Theorem 3.3, X is normal. □

Theorem 3.5. *In the class of semi-normal, almost β-normal spaces, the following statements are equivalent:*
(i) *X is normal.*
(ii) *X is densely normal.*
(iii) *X is weakly densely normal.*
(iv) *X is almost normal.*
(v) *X is κ-normal.*

Proof. The implications (i) $\Rightarrow$ (ii) $\Rightarrow$ (iii) $\Rightarrow$ (v) and (i) $\Rightarrow$ (iv) $\Rightarrow$ (v) are obvious from [5] and [15], respectively. To prove (v) $\Rightarrow$ (i), let X be a semi-normal, almost β-normal and κ-normal space. Then by Corollary 2.20 of [6], X is normal. □

4. Subspaces

Definition 4.1 ([9]). A subset A of a topological space X is said to be θ-embedded in X if every θ-closed set in the subspace topology of A is an intersection of A with a θ-closed set in X.

Example 4.2. θ-closed sets need not be θ-embedded. Let us consider closed unit interval $X = [0,1]$ with Smirnov's deleted sequence topology (see [17]). Let $A = K \cup \{0\}$, where $K = \{1/n\colon n \in \mathbb{N}\}$. Here A is θ-closed but not θ-embedded.

Theorem 4.3. *θ-embedded, θ-closed subspace of a wβ-normal space is wβ-normal.*

Proof. Let X be a wβ-normal space and Y be a θ-embedded, θ-closed set in X. Let A and B be two closed disjoint subsets of Y, out of which A is θ-closed. Since A is θ-closed and Y is θ-embedded, there exist a θ-closed set C in X such that $C \cap Y = A$ and a closed set $B = D \cap Y$, where D is closed in X. Since C and Y are θ-closed in X, $A = C \cap Y$ is θ-closed and $B = D \cap Y$ is closed in X, by wβ-normality of X, there exist open sets U and V in X such that $\overline{A \cap U} = A$, $\overline{B \cap V} = B$ and $\overline{U} \cap \overline{V} = \emptyset$. So $U \cap Y$ and $V \cap Y$ are two disjoint open sets in Y such that $\overline{A \cap U \cap Y} = A$, $\overline{B \cap V \cap Y} = B$ and $\overline{U \cap Y} \cap \overline{V \cap Y} = \emptyset$. Hence, Y is wβ-normal. □

Corollary 4.4. *Clopen subspace of a wβ-normal space is wβ-normal.*

In [8], it is observed that in the class of Hausdorff spaces, every θ-normal space is normal, and in the class of θ-regular spaces, every wθ-normal space is normal. Thus, it is natural to ask which wβ-normal spaces are normal. In Theorem 2.17 it is shown that in the class of regular spaces, every wβ-normal space is β-normal, and Theorems 3.1, 3.2 and 3.3 provide partial answer to the above question. But the following questions are still open.

Question 4.5. Which non-regular wβ-normal space are β-normal?

Question 4.6. Which non-regular wβ-normal space are normal?

It is well known that every compact Hausdorff space is normal and every regular Lindelöf space is normal. But in the absence of lower separation axioms, compactness need not imply normality. On the contrary, it is established that every paracompact space is θ-normal and every Lindelöf space (or almost compact space) is wθ-normal (see [8]). Thus, it is required to investigate the interrelation that exists among generalizations of compactness and wβ-normality.

Question 4.7. Which generalized notions of compactness is a subclass of the class of wβ-normal spaces?

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