ON WEAKENED (α,δ)-SKEW ARMENDARIZ RINGS

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Abstract. In this note, for a ring endomorphism α and an α-derivation δ of a ring R, the notion of weakened (α,δ)-skew Armendariz rings is introduced as a generalization of α-rigid rings and weak Armendariz rings. It is proved that R is a weakened (α,δ)-skew Armendariz ring if and only if $T_n(R)$ is weakened $(\overline{\alpha},\overline{\delta})$-skew Armendariz if and only if $R[x]/(x^n)$ is weakened $(\overline{\alpha},\overline{\delta})$-skew Armendariz ring for any positive integer n.

Keywords: Armendariz ring; (α,δ)-skew Armendariz ring; weak Armendariz ring; weak (α,δ)-skew Armendariz ring

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1. Introduction

Throughout this paper, R denotes an associative ring with unity, $\alpha\colon R\to R$ is an endomorphism and δ an α-derivation of R, that is, δ is an additive map such that $\delta(ab)=\delta(a)b+\alpha(a)\delta(b)$ for all $a,b\in R$. We denote by $R[x;\alpha,\delta]$ the Ore extension whose elements are the polynomials over R, the addition is defined as usual and the multiplication subject to the relation $xa=\alpha(a)x+\delta(a)$ for any $a\in R$. Rege and Chhawchharia in [22] introduced the notion of an Armendariz ring. They defined a ring R to be an Armendariz ring if whenever polynomials $f(x)=a_0+a_1x+\ldots+a_mx^m$, $g(x)=b_0+b_1x+\ldots+b_nx^n\in R[x]$ satisfy $f(x)g(x)=0$, then $a_ib_j=0$ for each i and j. The name "Armendariz ring" was chosen because Armendariz (see [5]) had noted that every reduced ring satisfies this condition. Some properties of Armendariz rings were studied in Rege and Chhawchharia [22], Armendariz [5], Anderson and Camillo [2], Huh et al. [14], and Kim and Lee [16]. Liu and Zhao in [20] called a ring R weak Armendariz if whenever polynomials $f(x)=a_0+a_1x+\ldots+a_mx^m$, $g(x)=b_0+b_1x+\ldots+b_nx^n\in R[x]$ satisfy $f(x)g(x)=0$, then $a_ib_j\in\operatorname{nil}(R)$

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for each i and j, where $\mathrm{nil}(R)$ denotes the set of all nilpotent elements of R. For an endomorphism α and an α-derivation δ of a ring R, Moussavi and Hashemi (see [21]) called R an (α,δ)-skew Armendariz ring if whenever polynomials $f(x) = a_0 + a_1x + \ldots + a_mx^m$, $g(x) = b_0 + b_1x + \ldots + b_nx^n \in R[x;\alpha,\delta]$ satisfy $f(x)g(x) = 0$, then $a_ix^ib_jx^j = 0$ for each i and j, which is a generalization of α-rigid rings and Armendariz rings. Alhevaz et al. in [1] called a ring R weak (α,δ)-skew Armendariz if whenever polynomials $f(x) = a_0 + a_1x + \ldots + a_mx^m$, $g(x) = b_0 + b_1x + \ldots + b_nx^n \in R[x;\alpha,\delta]$ satisfy $f(x)g(x) = 0$, then $a_ix^ib_jx^j \in \mathrm{nil}(R)[x;\alpha,\delta]$ for each i and j.

According to Krempa (see [17]), an endomorphism α of a ring R is said to be rigid if $a\alpha(a) = 0$ implies $a = 0$ for $a \in R$. Hong et al. in [13], Definition 3 called a ring R α-rigid if there exists a rigid endomorphism α of R. Note that any rigid endomorphism of a ring R is a monomorphism and α-rigid rings are reduced rings by Hong et al. (see [13]). Properties of α-rigid rings have been studied in Krempa [17], Hong et al. [13], and Hirano [11].

By [4], a ring R is α-compatible if for all $a, b \in R$, $ab = 0 \Leftrightarrow a\alpha(b) = 0$. In [10], Hashemi and Moussavi introduced (α,δ)-compatible rings and studied their properties. For an α-derivation δ of R, the ring is said to be δ-compatible if for each $a, b \in R$, $ab = 0 \Rightarrow a\delta(b) = 0$. A ring R is (α,δ)-compatible if it is both α-compatible and δ-compatible. In this case, clearly the endomorphism α is monomorphic. Also, any α-rigid ring is (α,δ)-compatible, see [13], Lemma 4.

For an endomorphism α and an α-derivation δ of a ring R, we call R a weakened (α,δ)-skew Armendariz ring if whenever polynomials $f(x) = \sum\limits_{i=0}^{m} a_ix^i$ and $g(x) = \sum\limits_{j=0}^{n} b_jx^j \in R[x;\alpha,\delta]$ satisfy $f(x)g(x) = 0$, then $a_ix^ib_jx^j \in \mathrm{nil}(R[x;\alpha,\delta])$ for each i and j. Clearly, weak Armendariz rings are weakened (α,δ)-skew Armendariz. We show that weakly 2-primal (α,δ)-compatible rings are weakened (α,δ)-skew Armendariz and thus weakened (α,δ)-skew Armendariz rings are a common generalization of α-rigid rings and weak Armendariz rings. Also, we prove that R is a weakened (α,δ)-skew Armendariz ring if and only if the $n \times n$ upper triangular matrix ring $T_n(R)$ is weakened $(\overline{\alpha},\overline{\delta})$-skew Armendariz if and only if $R[x]/(x^n)$ is weakened $(\overline{\alpha},\overline{\delta})$-skew Armendariz ring for any positive integer n.

2. Weakened (α,δ)-skew Armendariz rings

Let δ be an α-derivation of a ring R. For any $0 \leqslant u \leqslant v$ $(u, v \in \mathbb{N})$, $f_u^v \in \mathrm{End}(R,+)$ will denote the map which is the sum of all possible "words" in α, δ built with u letters α and $(v-u)$ letters δ. For instance, $f_2^4 = \alpha^2\delta^2 + \alpha\delta^2\alpha + \delta^2\alpha^2 + \alpha\delta\alpha\delta + \delta\alpha^2\delta + \delta\alpha\delta\alpha$. In particular, $f_0^0 = 1, f_0^n = \delta^n, \ldots, f_{n-1}^n = \alpha^{n-1}\delta + \alpha^{n-2}\delta\alpha + \ldots + \delta\alpha^{n-1}$

and $f_n^n = \alpha^n$, where $n \in \mathbb{N}$. For any positive integer n and $r \in R$ we have $x^n r = \sum\limits_{i=0}^{n} f_i^n(r)x^i$ in the ring $R[x;\alpha,\delta]$ (see [18], Lemma 4.1).

Definition 2.1. Let α be an endomorphism and δ an α-derivation of a ring R. The ring R is called a *weakened* (α,δ)*-skew Armendariz ring* if for each elements $f(x) = \sum\limits_{i=0}^{m} a_i x^i$ and $g(x) = \sum\limits_{j=0}^{n} b_j x^j \in R[x;\alpha,\delta]$, $f(x)g(x) = 0$ implies $a_i x^i b_j x^j \in \operatorname{nil}(R[x;\alpha,\delta])$ for each i and j.

Note that each Armendariz (or weak Armendariz) ring is weakened (α,δ)-skew Armendariz, where α is the identity endomorphism of R and δ is the zero mapping. The following example shows that there exists an endomorphism α and an α-derivation δ of an Armendariz (or weak Armendariz) ring R such that R is not weakened (α,δ)-skew Armendariz.

Example 2.2. Let S be a reduced ring and $R = S[x]$ a polynomial ring over S. Then R is reduced and so Armendariz (or weak Armendariz). Consider the endomorphism $\alpha\colon R \to R$ given by $\alpha(f(x)) = f(0)$ and α-derivation $\delta\colon R \to R$ by $\delta(f(x)) = xf(x) - f(0)x$. Take $p(y) = x - y$ and $q(y) = x + xy \in R[y;\alpha,\delta]$. Then $p(y)q(y) = 0$. But x^2 is not nilpotent and hence R is not weakened (α,δ)-skew Armendariz.

Clearly, every subring S with $\alpha(S) \subseteq S$ and $\delta(S) \subseteq S$ of a weakened (α,δ)-skew Armendariz ring is also weakened (α,δ)-skew Armendariz.

It will be useful to establish a criteria for transfering the weakened (α,δ)-skew Armendariz condition from one ring to another.

Proposition 2.3. *Let α be an endomorphism and δ an α-derivation of a ring R. Let S be a ring and $\gamma\colon R \to S$ a ring isomorphism. Then R is weakened (α,δ)-skew Armendariz if and only if S is weakened $(\gamma\alpha\gamma^{-1},\gamma\delta\gamma^{-1})$-skew Armendariz.*

Proof. Let $\alpha' = \gamma\alpha\gamma^{-1}$ and $\delta' = \gamma\delta\gamma^{-1}$. Clearly, α' is an endomorphism of S. Also $\delta'(ab) = \gamma\delta(\gamma^{-1}(a)\gamma^{-1}(b)) = \gamma((\delta\gamma^{-1})(a)\gamma^{-1}(b) + (\alpha\gamma^{-1})(a)(\delta\gamma^{-1})(b)) = \delta'(a)b + \alpha'(a)\delta'(b)$. Thus δ' is an α'-derivation on S. Suppose that $a' = \gamma(a)$ and $b' = \gamma(b)$ for each $a,b \in R$. Note that

$$\begin{aligned}\gamma(a\alpha^k\delta^t(b)) &= a'\gamma(\alpha^k\delta^t(b)) = a'\gamma(\alpha^k\gamma^{-1}\gamma\delta^t\gamma^{-1}\gamma(b))\\ &= a'(\gamma\alpha\gamma^{-1})^k(\gamma\delta\gamma^{-1})^t(b') = a'\alpha'^k\delta'^t(b').\end{aligned}$$

Therefore $\gamma(af_u^v(b)) = a'f_u^v(b')$ for each $a,b \in R$ and $0 \leqslant u \leqslant v$. Let $g(x) = \sum\limits_{i=0}^{m} a_i x^i$ and $h(x) = \sum\limits_{j=0}^{n} b_j x^j \in R[x;\alpha,\delta]$. According to the above argument, $g(x)h(x) = 0$

in $R[x;\alpha,\delta]$ if and only if $g'(x)h'(x)=0$ in $S[x;\alpha',\delta']$, where $g'(x)=\sum\limits_{i=0}^{m} a_i'x^i$ and $h'(x)=\sum\limits_{j=0}^{n} b_j'x^j \in S[x;\alpha',\delta']$. Also $a_ix^ib_jx^j \in \mathrm{nil}(R[x;\alpha,\delta])$ for each i, j if and only if $a_i'x^ib_j'x^j \in \mathrm{nil}(S[x;\alpha',\delta'])$ for each i, j. Thus, R is weakened (α,δ)-skew Armendariz if and only if S is weakened $(\gamma\alpha\gamma^{-1},\gamma\delta\gamma^{-1})$-skew Armendariz. □

Let α be an endomorphism and δ an α-derivation of a ring R. Recall that for an ideal I of R, if $\alpha(I)\subseteq I$, then $\overline{\alpha}\colon R/I\to R/I$ defined by $\overline{\alpha}(a+I)=\alpha(a)+I$ for $a\in R$ is an endomorphism of a factor ring R/I, and if $\delta(I)\subseteq I$, then $\overline{\delta}\colon R/I\to R/I$ defined by $\overline{\delta}(a+I)=\delta(a)+I$ for $a\in R$ is an $\overline{\alpha}$-derivation of a factor ring R/I. Also, for each $f(x)=\sum\limits_{i=0}^{m} a_ix^i \in R[x;\alpha,\delta]$, denote $\overline{f}(x)=\sum\limits_{i=0}^{m}\overline{a}_ix^i \in (R/I)[x;\overline{\alpha},\overline{\delta}]$, where $\overline{a}_i=a_i+I$ for each i.

Proposition 2.4. *Let α be an endomorphism and δ an α-derivation of a ring R. Let I be an ideal of R with $\alpha(I)\subseteq I$ and $\delta(I)\subseteq I$. If R/I is a weakened $(\overline{\alpha},\overline{\delta})$-skew Armendariz ring and $I[x;\alpha,\delta]$ is nil, then R is a weakened (α,δ)-skew Armendariz ring.*

P r o o f. Let $f(x)=\sum\limits_{i=0}^{m} a_ix^i$ and $g(x)=\sum\limits_{j=0}^{n} b_jx^j \in R[x;\alpha,\delta]$ satisfy $f(x)g(x)=0$. Then from canonical ring isomorphism $R[x;\alpha,\delta]/I[x;\alpha,\delta]\cong(R/I)[x;\overline{\alpha},\overline{\delta}]$ we have

$$\sum_{i=0}^{m}\overline{a}_ix^i\sum_{j=0}^{n}\overline{b}_jx^j=0.$$

Thus, $\overline{a}_ix^i\overline{b}_jx^j \in \mathrm{nil}((R/I)[x;\overline{\alpha},\overline{\delta}])$ for each i, j, since R/I is weakened $(\overline{\alpha},\overline{\delta})$-skew Armendariz, then $(a_ix^ib_jx^j)^{n_{ij}} \in I[x;\alpha,\delta])$ for a positive integer n_{ij}. Since $I[x;\alpha,\delta]$ is nil, $a_ix^ib_jx^j \in \mathrm{nil}(R[x;\alpha,\delta])$ for each i and j. Therefore R is weakened (α,δ)-skew Armendariz. □

Recall that a ring R is an NI ring if the set of nilpotent elements, $\mathrm{nil}(R)$, forms an ideal. In the following lemma, we determine a property for idempotents of a weakened (α,δ)-skew Armendariz NI ring.

Lemma 2.5. *Let R be a weakened (α,δ)-skew Armendariz NI ring. Then $\delta(e)\in \mathrm{nil}(R)$ for each $e^2=e\in R$.*

P r o o f. Let $e^2=e\in R$. Then we have $\delta(e)=\delta(e^2)=\delta(e)e+\alpha(e)\delta(e)$. Now suppose that $f(x)=\delta(e)+\alpha(e)x$ and $g(x)=(e-1)+(e-1)x\in R[x;\alpha,\delta]$. Then we have $f(x)g(x)=0$. Since R is a weakened (α,δ)-skew Armendariz ring, $\delta(e)(e-1)=\delta(e)e-\delta(e)\in \mathrm{nil}(R)$. On the other hand, if we take $p(x)=\delta(e)-(1-\alpha(e))x$ and $q(x)=e+ex\in R[x;\alpha,\delta]$, then we have $p(x)q(x)=0$. Thus, $\delta(e)e\in \mathrm{nil}(R)$ since R is a weakened (α,δ)-skew Armendariz ring. So $\delta(e)\in \mathrm{nil}(R)$, as desired. □

Recall that a ring R is Abelian if every idempotent of R is central. The following theorem is a characterization of an *Abelian* ring R to be weakened (α,δ)-skew Armendariz in terms of its idempotents.

Theorem 2.6. *Let R be an Abelian ring, α an endomorphism and δ an α-derivation of R. Then the following statements are equivalent:*

(i) *R is weakened (α,δ)-skew Armendariz;*

(ii) *For each idempotent $e\in R$ such that $\alpha(e)=e$ and $\delta(e)=0$, eR and $(1-e)R$ are weakened (α,δ)-skew Armendariz;*

(iii) *For an idempotent $e\in R$ such that $\alpha(e)=e$ and $\delta(e)=0$, eR and $(1-e)R$ are weakened (α,δ)-skew Armendariz.*

Proof. (i) $\Rightarrow$ (ii): It is obvious, since eR and $(1-e)R$ are subrings of R.

(ii) $\Rightarrow$ (iii): It is clear.

(iii) $\Rightarrow$ (i): Suppose that for an idempotent $e\in R$ such that $\alpha(e)=e$ and $\delta(e)=0$, eR and $(1-e)R$ are weakened (α,δ)-skew Armendariz and let $f(x)=\sum\limits_{i=0}^{m}a_ix^i$ and $g(x)=\sum\limits_{j=0}^{n}b_jx^j\in R[x;\alpha,\delta]$ with $f(x)g(x)=0$. Then $(ef(x))(eg(x))=0$ and $((1-e)f(x))((1-e)g(x))=0$. Since eR and $(1-e)R$ are weakened (α,δ)-skew Armendariz, there exist $m_{ij},n_{ij}\in\mathbb{N}$ such that $(ea_ix^ieb_jx^j)^{m_{ij}}=0$ and $((1-e)a_ix^i(1-e)b_jx^j)^{n_{ij}}=0$. On the other hand, since $\alpha(e)=e$ and $\delta(e)=0$, we have $\alpha(eb_j)=e\alpha(b_j)$ and $\delta(eb_j)=e\delta(b_j)$. Hence, one can see that $(ea_ix^ieb_jx^j)^{m_{ij}}=e(a_ix^ib_jx^j)^{m_{ij}}=0$ and $((1-e)a_ix^i(1-e)b_jx^j)^{n_{ij}}=(1-e)(a_ix^ib_jx^j)^{n_{ij}}=0$. Let $k_{ij}=\max\{m_{ij},n_{ij}\}$. Then $e(a_ix^ib_jx^j)^{k_{ij}}=0$ and $(1-e)(a_ix^ib_jx^j)^{k_{ij}}=0$. Therefore $(a_ix^ib_jx^j)^{k_{ij}}=e(a_ix^ib_jx^j)^{k_{ij}}+(1-e)(a_ix^ib_jx^j)^{k_{ij}}=0$. Hence R is weakened (α,δ)-skew Armendariz. $\square$

For weak Armendariz rings we have the following result.

Proposition 2.7. *If $R[x;\alpha]$ is a weak Armendariz ring, then R is a weakened α-skew Armendariz ring.*

Proof. Suppose $f(x)=\sum\limits_{i=0}^{m}a_ix^i$ and $g(x)=\sum\limits_{j=0}^{n}b_jx^j\in R[x;\alpha]$ satisfy $f(x)g(x)=0$. Then we have $c_k=a_0b_k+a_1\alpha(b_{k-1})+\ldots+a_k\alpha^k(b_0)=0$ for each $0\leqslant k\leqslant m+n$. Now, let

$$p(y)=a_0+(a_1x)y+(a_2x^2)y^2+\ldots+(a_mx^m)y^m,$$
$$q(y)=b_0+(b_1x)y+(b_2x^2)y^2+\ldots+(b_nx^n)y^n$$

be polynomials in $R[x;\alpha][y]$. Thus, we have $p(y)q(y) = \sum\limits_{k=0}^{m+n}(c_kx^k)y^k = 0$, since $c_k = 0$ for each $0 \leqslant k \leqslant m+n$. So $a_ix^ib_jx^j \in \operatorname{nil}(R[x;\alpha])$ for each $0 \leqslant i \leqslant m$ and $0 \leqslant j \leqslant n$, since $R[x;\alpha]$ is weak Armendariz. Hence, R is a weakened α-skew Armendariz ring and the result follows. $\square$

Let α_i be an endomorphism and δ_i an α_i-derivation of a ring R_i, $i = 1, 2, \ldots, k$. Let $R = \bigoplus\limits_{i=1}^{k} R_i$. Then the map $\alpha\colon R \to R$ defined by $\alpha((a_i)) = (\alpha_i(a_i))$ is an endomorphism of R and $\delta\colon R \to R$ defined by $\delta((a_i)) = (\delta_i(a_i))$ is an α-derivation of R.

Proposition 2.8. *Let α_i be an endomorphism and δ_i an α_i-derivation of a ring R_i for each $1 \leqslant i \leqslant k$. Then R_i is a weakened (α_i, δ_i)-skew Armendariz ring if and only if $R = \bigoplus\limits_{i=1}^{k} R_i$ is a weakened (α, δ)-skew Armendariz ring.*

Proof. It is not hard to see that there exists a ring isomorphism $\varphi\colon R[x;\alpha,\delta] \to \bigoplus\limits_{i=1}^{k}(R_i[x;\alpha_i,\delta_i])$, given by $\varphi\Big(\sum\limits_{s=0}^{m} A_sx^s\Big) = (f_i)$, where $A_s = (a_{1s}, a_{2s}, \ldots, a_{ks})$ in R and $f_i(x) = \sum\limits_{s=0}^{m} a_{is}x^s$ in $R_i[x;\alpha_i,\delta_i]$ for each $0 \leqslant s \leqslant m$ and $1 \leqslant i \leqslant k$. Let $f(x) = \sum\limits_{s=0}^{m} A_sx^s$ and $g(x) = \sum\limits_{t=0}^{n} B_tx^t \in R[x;\alpha,\delta]$ satisfy $f(x)g(x) = 0$, where $A_s = (a_{1s}, a_{2s}, \ldots, a_{ks})$ and $B_t = (b_{1t}, b_{2t}, \ldots, b_{kt}) \in R$ and $a_{is}, b_{it} \in R_i$ for each $0 \leqslant s \leqslant m$ and $0 \leqslant t \leqslant n$. Then from isomorphism $R[x;\alpha,\delta] \cong \bigoplus\limits_{i=1}^{k}(R_i[x;\alpha_i,\delta_i])$ we have that $f_i(x)g_i(x) = 0$ for each $1 \leqslant i \leqslant k$, where $f_i(x) = \sum\limits_{s=0}^{m} a_{is}x^s$ and $g_i(x) = \sum\limits_{t=0}^{n} b_{it}x^t \in R_i[x;\alpha_i,\delta_i]$. Since R_i is weakened (α_i,δ_i)-skew Armendariz for every $1 \leqslant i \leqslant k$, there exists $p_{sti} \in \mathbb{N}$ such that $(a_{is}x^sb_{it}x^t)^{p_{sti}} = 0$ for each $1 \leqslant i \leqslant k$. Let $p_{st} = \max\{p_{st1}, p_{st2}, \ldots, p_{stk}\}$. Then $(A_sx^sB_tx^t)^{p_{st}} = 0$. Therefore $R = \bigoplus\limits_{i=1}^{k} R_i$ is a weakened (α,δ)-skew Armendariz ring. Conversely, since R_i is an invariant subring of R for each $1 \leqslant i \leqslant k$, the assertion holds. $\square$

Let R be a ring and σ denotes an endomorphism of R with $\sigma(1) = 1$. In [6] the authors introduced *skew triangular matrix* ring, denoted by $T_n(R,\sigma)$, as a set of all triangular matrices with addition point-wise and a new multiplication subject to the condition $E_{ij}r = \sigma^{j-i}(r)E_{ij}$. So $(a_{ij})(b_{ij}) = (c_{ij})$, where $c_{ij} = a_{ii}b_{ij} + a_{i,i+1}\sigma(b_{i+1,j}) + \ldots + a_{ij}\sigma^{j-i}(b_{jj})$ for each $i \leqslant j$.

The subring of the skew triangular matrices with constant main diagonal is denoted by $S(R,n,\sigma)$; and the subring of the skew triangular matrices with constant diagonals is denoted by $T(R,n,\sigma)$. We can denote $A = (a_{ij}) \in T(R,n,\sigma)$ by $(a_{11}, \ldots, a_{nn})$. Then $T(R,n,\sigma)$ is a ring with addition point-wise and multiplication given by:

$(a_0,\ldots,a_{n-1})(b_0,\ldots,b_{n-1})=(a_0b_0, a_0*b_1+a_1*b_0,\ldots,a_0*b_{n-1}+\ldots+a_{n-1}*b_0)$, with $a_i*b_j=a_i\sigma^i(b_j)$ for each i and j. Therefore, clearly one can see that $T(R,n,\sigma)\cong R[x;\sigma]/(x^n)$, where (x^n) is the ideal generated by x^n in $R[x;\sigma]$.

Also, we consider the following two subrings of $S(R,n,\sigma)$:

$$A(R,n,\sigma)=\Bigg\{\sum_{j=1}^{\lfloor\frac{n}{2}\rfloor}\sum_{i=1}^{n-j+1}a_jE_{i,i+j-1}+\sum_{j=\lfloor\frac{n}{2}\rfloor+1}^{n}\sum_{i=1}^{n-j+1}a_{i,i+j-1}E_{i,i+j-1}\Bigg\};$$
$$B(R,n,\sigma)=\{A+rE_{1k}\colon\ A\in A(R,n,\sigma)\text{ and }r\in R\},\quad n=2k\geqslant 4.$$

Let σ be an endomorphism of a ring R, α an endomorphism of R and δ an α-derivation of R such that $\sigma\alpha=\alpha\sigma$ and $\delta\sigma=\sigma\delta$. The endomorphism α of R is extended to the endomorphism $\overline{\alpha}\colon D\to D$ defined by $\overline{\alpha}((a_{ij}))=(\alpha(a_{ij}))$ and the α-derivation δ of R is also extended to $\bar{\delta}\colon D\to D$ defined by $\bar{\delta}((a_{ij}))=(\delta(a_{ij}))$, where D is one of the rings $T_n(R,\sigma)$, $S(R,n,\sigma)$, $A(R,n,\sigma)$, $B(R,n,\sigma)$ or $T(R,n,\sigma)$. Also, the map $\overline{\sigma}\colon R[x;\alpha,\delta]\to R[x;\alpha,\delta]$ defined by $\overline{\sigma}\Big(\sum\limits_{i=0}^{m}a_ix^i\Big)=\sum\limits_{i=0}^{m}\sigma(a_i)x^i$ is an endomorphism of $R[x;\alpha,\delta]$.

Kim and Lee in [15], Example 1 showed that $n\times n$ upper triangular matrix rings over a ring R are not Armendariz when $n\geqslant 2$. But we have the following result.

Proposition 2.9. *Let σ and α be endomorphisms of a ring R and δ an α-derivation of R such that $\sigma\alpha=\alpha\sigma$, $\delta\sigma=\sigma\delta$ and n is a positive integer number. Then R is a weakened (α,δ)-skew Armendariz ring if and only if D is a weakened $(\overline{\alpha},\bar{\delta})$-skew Armendariz ring, where D is one of the rings $T_n(R,\sigma)$, $S(R,n,\sigma)$, $A(R,n,\sigma)$, $B(R,n,\sigma)$, $T(R,n,\sigma)$.*

Proof. We only prove this proposition for the case $D=T_n(R,\sigma)$. Note that any invariant subring of weakened (α,δ)-skew Armendariz rings is a weakened (α,δ)-skew Armendariz ring. Thus, if $T_n(R,\sigma)$ is a weakened $(\overline{\alpha},\bar{\delta})$-skew Armendariz ring, then R is a weakened (α,δ)-skew Armendariz ring. Conversely, Let $I=\{A\in D\colon$ each diagonal entry of A is zero$\}$. Then $I[x;\overline{\alpha},\bar{\delta}]$ is a nil ideal of $D[x;\overline{\alpha},\bar{\delta}]$. On the other hand, we can obtain $D/I\cong\bigoplus\limits_{i=1}^{n}R_i$, where $R_i=R$. The proof is completed by Proposition 2.4 and Proposition 2.8. □

Corollary 2.10. *If R is an (α,δ)-skew Armendariz ring, then D is a weakened $(\overline{\alpha},\bar{\delta})$-skew Armendariz ring, where D is one of the rings $T_n(R,\sigma)$, $S(R,n,\sigma)$, $A(R,n,\sigma)$, $B(R,n,\sigma)$, $T(R,n,\sigma)$.*

Given a ring R and a bimodule ${}_R\mathrm{M}_R$, the trivial extension of R by M is $T(R,M)=R\oplus M$ with the usual addition and the multiplication: $(r_1,m_1)(r_2,m_2)=(r_1r_2, r_1m_2+m_1r_2)$.

This is isomorphic to the ring of all matrices $\begin{pmatrix} r & m \\ 0 & r \end{pmatrix}$, where $r \in R$ and $m \in M$ and the usual matrix operations are used.

Corollary 2.11. *Let α be an endomorphism and δ an α-derivation of a ring R. Then R is a weakened (α, δ)-skew Armendariz ring if and only if the trivial extension $T(R, R)$ is a weakened $(\overline{\alpha}, \overline{\delta})$-skew Armendariz ring.*

Proof. It follows from Proposition 2.9. □

Note that if σ is an identity endomorphism of R, then we have the following corollary.

Corollary 2.12. *Let σ and α be endomorphisms of a ring R and δ an α-derivation of R such that $\sigma\alpha = \alpha\sigma$ and $\delta\sigma = \sigma\delta$. Then we have the following statements:*

(i) *R is a weakened (α, δ)-skew Armendariz ring if and only if for each positive integer n, $R[x; \sigma]/(x^n)$ is a weakened $(\overline{\alpha}, \overline{\delta})$-skew Armendariz ring.*

(ii) *R is a weakened (α, δ)-skew Armendariz ring if and only if for each positive integer n, $R[x]/(x^n)$ is a weakened $(\overline{\alpha}, \overline{\delta})$-skew Armendariz ring.*

Now we can give the examples of weakened (α, δ)-skew Armendariz rings which are not (α, δ)-skew Armendariz.

Example 2.13. Let α be an endomorphism and δ an α-derivation of a field F and $R = \begin{pmatrix} F & F \\ 0 & F \end{pmatrix}$ be the 2-by-2 upper triangular matrix ring over F. Let $f(x) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix} x$ and $g(x) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} x \in R[x; \overline{\alpha}, \overline{\delta}]$. Then it is easy to see that $f(x)g(x) = 0$, but $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} x \neq 0$. So R is not $(\overline{\alpha}, \overline{\delta})$-skew Armendariz. Since F is a field, F is an (α, δ)-skew Armendariz. Thus, by Corollary 2.10, R is a weakened $(\overline{\alpha}, \overline{\delta})$-skew Armendariz ring.

Example 2.14. Let R be a weakened (α, δ)-skew Armendariz ring. Let

$$S_n = \left\{ \begin{pmatrix} a & a_{12} & a_{13} & \dots & a_{1n} \\ 0 & a & a_{23} & \dots & a_{2n} \\ 0 & 0 & a & \dots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & a \end{pmatrix} : a, a_{ij} \in R \right\}$$

with $n \geqslant 4$.

Suppose

$$f(x)=\begin{pmatrix}0&1&0&\dots&0\\0&0&0&\dots&0\\0&0&0&\dots&0\\\vdots&\vdots&\vdots&\ddots&\vdots\\0&0&0&\dots&0\end{pmatrix}+\begin{pmatrix}0&1&-1&0&\dots&0\\0&0&0&0&\dots&0\\0&0&0&0&\dots&0\\\vdots&\vdots&\vdots&\vdots&\ddots&\vdots\\0&0&0&0&\dots&0\end{pmatrix}x$$

and

$$g(x)=\begin{pmatrix}0&0&0&\dots&0\\0&0&0&\dots&0\\0&0&0&\dots&1\\0&0&0&\dots&0\\\vdots&\vdots&\vdots&\ddots&\vdots\\0&0&0&\dots&0\end{pmatrix}+\begin{pmatrix}0&0&0&\dots&0\\0&0&0&\dots&1\\0&0&0&\dots&1\\0&0&0&\dots&0\\\vdots&\vdots&\vdots&\ddots&\vdots\\0&0&0&\dots&0\end{pmatrix}x$$

be polynomials in $S_n[x;\overline{\alpha},\overline{\delta}]$. Then it is easy to see that $f(x)g(x)=0$, but

$$\begin{pmatrix}0&1&0&\dots&0\\0&0&0&\dots&0\\0&0&0&\dots&0\\\vdots&\vdots&\vdots&\ddots&\vdots\\0&0&0&\dots&0\end{pmatrix}\begin{pmatrix}0&0&0&\dots&0\\0&0&0&\dots&1\\0&0&0&\dots&1\\0&0&0&\dots&0\\\vdots&\vdots&\vdots&\ddots&\vdots\\0&0&0&\dots&0\end{pmatrix}x\neq 0.$$

So S_n is not $(\overline{\alpha},\overline{\delta})$-skew Armendariz, but S_n is a weakened $(\overline{\alpha},\overline{\delta})$-skew Armendariz ring by Proposition 2.9, since any subring of weakened $(\overline{\alpha},\overline{\delta})$-skew Armendariz rings is a weakened $(\overline{\alpha},\overline{\delta})$-skew Armendariz ring.

From Proposition 2.9, one may suspect that if R is weakened (α,δ)-skew Armendariz, then every $n\times n$ full matrix ring $M_n(R)$ over R is a weakened $(\overline{\alpha},\overline{\delta})$-skew Armendariz ring, where $n\geqslant 2$. But the following example erases this possibility.

Example 2.15. Let α be an endomorphism and δ an α-derivation of a ring R and R be a weakened (α,δ)-skew Armendariz ring. Let $S=M_2(R)$. Suppose

$$f(x)=\begin{pmatrix}0&1\\0&0\end{pmatrix}+\begin{pmatrix}1&0\\0&0\end{pmatrix}x\quad\text{and}\quad g(x)=\begin{pmatrix}1&1\\0&0\end{pmatrix}+\begin{pmatrix}0&0\\-1&-1\end{pmatrix}x$$

be polynomials in $S[x;\overline{\alpha},\overline{\delta}]$. Then it is easy to see that $f(x)g(x)=0$, but

$$\begin{pmatrix}1&0\\0&0\end{pmatrix}x\begin{pmatrix}1&1\\0&0\end{pmatrix}=\begin{pmatrix}1&1\\0&0\end{pmatrix}x$$

is not nilpotent. Thus, S is not weakened $(\overline{\alpha},\overline{\delta})$-skew Armendariz.

Let D be a ring and C a subring of D with $1_D \in C$. With addition and multiplication defined component-wise,

$$R = \Re(D, C) = \{(d_1, \ldots, d_n, c, c, \ldots)\colon\ d_i \in D,\ c \in C,\ n \geqslant 1\}$$

is a ring (see [7]). For an endomorphism α and an α-derivation δ of D such that $\alpha(C) \subseteq C$ and $\delta(C) \subseteq C$, the natural extension $\overline{\alpha}\colon R \to R$ defined by

$$\overline{\alpha}((d_1, \ldots, d_n, c, c, \ldots)) = (\alpha(d_1), \ldots, \alpha(d_n), \alpha(c), \alpha(c), \ldots)$$

for $(d_1, \ldots, d_n, c, c, \ldots) \in R$ is an endomorphism of R and $\bar{\delta}\colon R \to R$ defined by $\bar{\delta}((d_1, \ldots, d_n, c, c, \ldots)) = (\delta(d_1), \ldots, \delta(d_n), \delta(c), \delta(c), \ldots)$ for $(d_1, \ldots, d_n, c, c, \ldots) \in R$ is an $\overline{\alpha}$-derivation of R.

Theorem 2.16. *Let α be an endomorphism and δ an α-derivation of a ring D and let C be a subring of D with $1_D \in C$, $\alpha(C) \subseteq C$ and $\delta(C) \subseteq C$. Then D is a weakened (α, δ)-skew Armendariz ring if and only if $R = \Re(D, C)$ is a weakened $(\overline{\alpha}, \bar{\delta})$-skew Armendariz ring.*

Proof. Suppose that D is a weakened (α, δ)-skew Armendariz ring. Let $f(x) = \sum\limits_{i=0}^{p} \xi_i x^i$ and $g(x) = \sum\limits_{j=0}^{q} \eta_j x^j \in R[x; \overline{\alpha}, \bar{\delta}]$ and $f(x)g(x) = 0$. Without loss of generality, we can assume that there exists a positive integer n such that $\xi_i = (a_{1i}, \ldots, a_{ni}, c_i, c_i, \ldots)$, $\eta_j = (b_{1j}, \ldots, b_{nj}, d_j, d_j, \ldots) \in R$ for all i, j. Let $f_s(x) = \sum\limits_{i=0}^{p} a_{si} x^i$, $g_s(x) = \sum\limits_{j=0}^{q} b_{sj} x^j$ with $1 \leqslant s \leqslant n$ and $f'(x) = \sum\limits_{i=0}^{p} c_i x^i$, $g'(x) = \sum\limits_{j=0}^{q} d_j x^j$. From $f(x)g(x) = 0$ we obtain $f_s(x)g_s(x) = 0$ and $f'(x)g'(x) = 0$ in $D[x; \alpha, \delta]$ for all s. Thus, $a_{si}x^i b_{sj}x^j \in \operatorname{nil}(D[x; \alpha, \delta])$ and $c_i x^i d_j x^j \in \operatorname{nil}(D[x; \alpha, \delta])$ for all i, j, s since D is weakened (α, δ)-skew Armendariz. Hence, there exist $t_{sij}, t'_{ij} \in \mathbb{N}$ such that $(a_{si}x^i b_{sj}x^j)^{t_{sij}} = 0$ and $(c_i x^i d_j x^j)^{t'_{ij}} = 0$ for $1 \leqslant s \leqslant n$. Let $t_{ij} = \max\{t_{1ij}, \ldots, t_{nij}, t'_{ij}\}$. Then we have $(\xi_i x^i \eta_j x^j)^{t_{ij}} = 0$ for all i, j. Therefore R is weakened $(\overline{\alpha}, \bar{\delta})$-skew Armendariz. Conversely, since D is an invariant subring of R, the assertion holds. $\square$

3. Weakly 2-primal (α,δ)-compatible rings and weakened (α,δ)-skew Armendariz rings

A ring R is *semicommutative* if the right annihilator of each element of R is an ideal (equivalently, if for all $a,b\in R$ we have $ab=0\Rightarrow aRb=0$). A ring R is *symmetric* if for all $a,b,c\in R$ we have $abc=0\Rightarrow bac=0$. A ring R is called *reversible* if for all $a,b\in R$ we have $ab=0\Rightarrow ba=0$. Recall that a ring R is 2-*primal* if $\mathrm{nil}(R)=\mathrm{Nil}_*(R)$, where $\mathrm{Nil}_*(R)$ denotes the prime radical of R. Hong et al. (see [12]) called a ring R to be *locally* 2-*primal* if each finite subset generates a 2-primal ring. Chen and Cui (see [8]) called a ring R *weakly* 2-*prima*l if the set of nilpotent elements in R coincides with its locally nipotent radical. Note that every reduced ring is symmetric by [3], Theorem 1.3, every symmetric ring is reversible, every reversible ring is semicommutative by [19], Proposition 1.3, every semicommutative ring is 2-primal by [23], Theorem 1.5, every 2-primal ring is locally 2-primal and every locally 2-primal ring is weakly 2-primal.

The following example shows that there exists a semicommutative ring with an endomorphism α and an α-derivation δ which is not weakened (α,δ)-skew Armendariz.

E x a m p l e 3.1. Let S be a reduced ring and $R=S[x]$ a polynomial ring over S. Then R is reduced and so semicommutative. Consider the endomorphism $\alpha\colon R\to R$ given by $\alpha(f(x))=f(0)$ and α-derivation $\delta\colon R\to R$ by $\delta(f(x))=xf(x)-f(0)x$. Take $p(y)=x-y$ and $q(y)=x+xy\in R[y;\alpha,\delta]$. Then $p(y)q(y)=0$. But x^2 is not nilpotent and hence R is not weakened (α,δ)-skew Armendariz.

The following example shows that weakened (α,δ)-skew Armendariz rings may not be semicommutative.

E x a m p l e 3.2. Let α be an endomorphism and δ an α-derivation of a division ring F and $R=\begin{pmatrix} F & F\\ 0 & F\end{pmatrix}$ be the 2-by-2 upper triangular matrix ring over F. Then R is not semicommutative by [14], Example 5. But by Corollary 2.10, R is a weakened $(\overline{\alpha},\overline{\delta})$-skew Armendariz ring.

Habibi and Moussavi (see [9]) called a *ring R nil (α,δ)-skew Armendariz* if whenever polynomials $f(x)=a_0+a_1x+\ldots+a_mx^m$, $g(x)=b_0+b_1x+\ldots+b_nx^n\in R[x;\alpha,\delta]$ satisfy $f(x)g(x)\in\mathrm{nil}(R)[x;\alpha,\delta]$, then $a_ix^ib_jx^j\in\mathrm{nil}(R)[x;\alpha,\delta]$ for each i and j.

Proposition 3.3. *Let R be an α-compatible ring such that* $\mathrm{nil}(R[x;\alpha,\delta])=\mathrm{nil}(R)[x;\alpha,\delta]$. *Then R is a weakened (α,δ)-skew Armendariz ring.*

P r o o f. Let $f(x)=\sum\limits_{i=0}^{m}a_ix^i$ and $g(x)=\sum\limits_{j=0}^{n}b_jx^j\in R[x;\alpha,\delta]$ such that $f(x)g(x)=0$. Since R is an α-compatible and $\mathrm{nil}(R[x;\alpha,\delta])=\mathrm{nil}(R)[x;\alpha,\delta]$, R is nil (α,δ)-skew Armendariz by [9], Proposition 2.9. Then $a_ix^ib_jx^j \in \operatorname{nil}(R)[x;\alpha,\delta]$ for each i, j. Hence $a_ix^ib_jx^j \in \operatorname{nil}(R[x;\alpha,\delta])$ for each i, j. Therefore R is a weakened (α,δ)-skew Armendariz ring. □

Wang et al. in [24], Corollary 2.1 proved that if R is a weakly 2-primal (α,δ)-compatible ring, then $\operatorname{nil}(R[x;\alpha,\delta]) = \operatorname{nil}(R)[x;\alpha,\delta]$. So we have the following result.

Proposition 3.4. *Every weakly 2-primal (α,δ)-compatible ring is weakened (α,δ)-skew Armendariz.*

Corollary 3.5. *α-rigid rings are weakened (α,δ)-skew Armendariz rings.*

The following example shows that the converse of Corollary 3.5 is not true in general.

Example 3.6. Let δ be an α-derivation of a ring R and R be an α-rigid ring. Then by Corollary 3.5, R is a weakened (α,δ)-skew Armendariz ring. Consider the following subring of $T_3(R)$:

$$R_3 = \left\{ \begin{pmatrix} a & b & c \\ 0 & a & d \\ 0 & 0 & a \end{pmatrix} : a,b,c,d \in R \right\}.$$

The endomorphism α of R is extended to the endomorphism $\overline{\alpha}\colon R_3 \to R_3$ defined by $\overline{\alpha}((a_{ij})) = (\alpha(a_{ij}))$ and the α-derivation δ of R is also extended to $\overline{\delta}\colon R_3 \to R_3$ defined by $\overline{\delta}((a_{ij})) = (\delta(a_{ij}))$. By Proposition 2.9, R_3 is weakened $(\overline{\alpha},\overline{\delta})$-skew Armendariz. But it is not $\overline{\alpha}$-rigid, by [10], Example 1.2.

Lemma 3.7. *Let α be an endomorphism and δ an α-derivation of a ring R. Then R is (α,δ)-compatible and reduced if and only if $R[x]$ is (α,δ)-compatible and reduced.*

Proof. We know, R is reduced if and only if $R[x]$ is reduced. Let R be (α,δ)-compatible and reduced. Let $f(x) = \sum\limits_{i=0}^{m} a_ix^i$ and $g(x) = \sum\limits_{j=0}^{n} b_jx^j \in R[x]$ with $f(x)g(x) = 0$. Since R is Armendariz, $a_ib_j = 0$ for all $0 \leqslant i \leqslant m$ and $0 \leqslant j \leqslant n$. Then $a_i\alpha(b_j) = 0, a_i\delta(b_j) = 0$ because R is (α,δ)-compatible. Thus

$$f(x)\alpha(g(x)) = \sum_{i=0}^{m} a_ix^i \sum_{j=0}^{n} \alpha(b_j)x^j = \sum_{k=0}^{m+n} \left(\sum_{i+j=k} a_i\alpha(b_j) \right) x^k = 0$$

and

$$f(x)\delta(g(x)) = \sum_{i=0}^{m} a_ix^i \sum_{j=0}^{n} \delta(b_j)x^j = \sum_{k=0}^{m+n} \left(\sum_{i+j=k} a_i\delta(b_j) \right) x^k = 0.$$

Now assume that $f(x)\alpha(g(x)) = 0$. Then we have

$$f(x)\alpha(g(x)) = \sum_{i=0}^{m} a_i x^i \sum_{j=0}^{n} \alpha(b_j) x^j = 0.$$

Since R is Armendariz, $a_i\alpha(b_j) = 0$ for all $0 \leqslant i \leqslant m$ and $0 \leqslant j \leqslant n$. So $a_i b_j = 0$ because R is (α, δ)-compatible. Hence

$$f(x)g(x) = \sum_{i=0}^{m} a_i x^i \sum_{j=0}^{n} b_j x^j = \sum_{k=0}^{m+n} \Big(\sum_{i+j=k} a_i b_j x^k \Big) = 0.$$

Therefore $R[x]$ is an (α, δ)-compatible ring. Conversely, it is clear. □

Proposition 3.8. *Let R be an (α, δ)-compatible and reduced ring. Then $R[x]$ is a weakened (α, δ)-skew Armendariz ring.*

P r o o f. Let R be an (α, δ)-compatible and reduced ring. Then $R[x]$ is (α, δ)-compatible and reduced, by Lemma 3.7. But every reduced ring is weakly 2-primal. Thus, $R[x]$ is a weakly 2-primal (α, δ)-compatible ring. Therefore $R[x]$ is a weakened (α, δ)-skew Armendariz ring, by Proposition 3.4. □

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