INITIAL MACLAURIN COEFFICIENT ESTIMATES FOR λ-PSEUDO-STARLIKE BI-UNIVALENT FUNCTIONS ASSOCIATED WITH SAKAGUCHI-TYPE FUNCTIONS

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Abstract. We introduce and study two certain classes of holomorphic and bi-univalent functions associating λ-pseudo-starlike functions with Sakaguchi-type functions. We determine upper bounds for the Taylor–Maclaurin coefficients $|a_2|$ and $|a_3|$ for functions belonging to these classes. Further we point out certain special cases for our results.

Keywords: holomorphic function; bi-univalent function; coefficient estimates; λ-pseudo-starlike function; Sakaguchi-type function

MSC 2020: 30C45, 30C50

1. Introduction

Denote by $\mathcal{A}$ the collection of all holomorphic functions in the unit disc $U = \{z \in \mathbb{C}\colon |z| < 1\}$ that have the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k. \tag{1.1}$$

Further, assume that S stands for the sub-collection of the set $\mathcal{A}$ consisting of functions in U satisfying (1.1) which are univalent in U.

Frasin (see [5]) introduced and studied the class $S(\gamma, m, n)$ consisting of functions $f \in \mathcal{A}$ which satisfy the condition

$$\mathrm{Re}\Big\{\frac{(m-n)zf'(z)}{f(mz)-f(nz)}\Big\} > \gamma,$$

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for some $0 \leqslant \gamma < 1$, $m, n \in \mathbb{C}$ with $m \neq n$, $|m| \leqslant 1$, $|n| \leqslant 1$ and for all $z \in U$. We note that the class $S(\gamma, 1, n)$ was studied by Owa et al. (see [13]), while the class $S(\gamma, 1, -1) \equiv S_s(\gamma)$ was considered by Sakaguchi (see [14]) and is called the Sakaguchi function of order γ. Also, $S(0, 1, -1) \equiv S_s$ is the class of starlike functions with respect to symmetrical points in U, and $S(\gamma, 1, 0) \equiv S^*(\gamma)$ is the class of starlike functions of order γ, $0 \leqslant \gamma < 1$.

In [2] Babalola defined the class $\mathcal{L}_\lambda(\gamma)$ of λ-pseudo-starlike functions of order γ which are the functions $f \in \mathcal{A}$ such that

$$\operatorname{Re}\Big\{\frac{z(f'(z))^\lambda}{f(z)}\Big\} > \gamma,$$

where $0 \leqslant \gamma < 1$, $\lambda \geqslant 1$, and $z \in U$. In particular, Babalola (see [2]) showed that all λ-pseudo-starlike functions are Bazilevič of type $1 - 1/\lambda$ and order $\gamma^{1/\lambda}$ and are univalent in U. It is observed that for $\lambda = 1$, we have the class of starlike functions.

According to the Koebe one-quarter theorem (see [4]) "every function $f \in S$ has an inverse f^{-1} which satisfies $f^{-1}(f(z)) = z$, $(z \in U)$ and $f(f^{-1}(w)) = w$, $|w| < r_0(f)$, $r_0(f) \geqslant \frac{1}{4}$", where

$$g(w) = f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + \ldots \tag{1.2}$$

For $f \in \mathcal{A}$, if both f and f^{-1} are univalent in U, we say that f is a bi-univalent function in U. We denote by Σ the class of bi-univalent functions in U given by (1.1). In fact, Srivastava et al. (see [20]) have revived the study of holomorphic and bi-univalent functions in recent years. Some examples of functions in the class Σ are

$$\frac{z}{1-z}, \quad \frac{1}{2}\log\Big(\frac{1+z}{1-z}\Big) \quad \text{and} \quad -\log(1-z)$$

with the corresponding inverse functions

$$\frac{w}{1+w}, \quad \frac{\mathrm{e}^{2w}-1}{\mathrm{e}^{2w}+1} \quad \text{and} \quad \frac{\mathrm{e}^{w}-1}{\mathrm{e}^{w}},$$

respectively. Conversely, examples of common functions that are not in Σ are

$$z - \frac{z^2}{2} \quad \text{and} \quad \frac{z}{1-z^2}.$$

Many researchers (see, for example, [1], [6], [7], [10], [15]–[19], [21]–[24]) have recently introduced and investigated several interesting subclasses of the bi-univalent function class Σ and they have found non-sharp estimates on the first two Taylor–Maclaurin coefficients $|a_2|$ and $|a_3|$.

We require the following lemma that will be used to prove our main results.

Lemma 1.1 ([4]). *If $h \in \mathcal{P}$, then $|c_k| \leqslant 2$ for each $k \in \mathbb{N}$, where $\mathcal{P}$ is the class of all functions h holomorphic in U for which*

$$\operatorname{Re}(h(z)) > 0, \quad z \in U,$$

where

$$h(z) = 1 + c_1 z + c_2 z^2 + \ldots, \quad z \in U.$$

2. Coefficient estimates for the function class $V_\Sigma(\delta, \lambda, m, n; \alpha)$

Definition 2.1. A function $f \in \Sigma$ given by (1.1) is said to be in the class $V_\Sigma(\delta, \lambda, m, n; \alpha)$ if the following conditions are satisfied:

$$\left|\arg\Big((1-\delta)\frac{(m-n)z(f'(z))^\lambda}{f(mz)-f(nz)} + \delta\frac{(m-n)((zf'(z))')^\lambda}{(f(mz)-f(nz))'}\Big)\right| < \frac{\alpha\pi}{2} \tag{2.1}$$

and

$$\left|\arg\Big((1-\delta)\frac{(m-n)w(g'(w))^\lambda}{g(mw)-g(nw)} + \delta\frac{(m-n)((wg'(w))')^\lambda}{(g(mw)-g(nw))'}\Big)\right| < \frac{\alpha\pi}{2}, \tag{2.2}$$

where $0 < \alpha \leqslant 1$, $0 \leqslant \delta \leqslant 1$, $\lambda \geqslant 1$, $m, n \in \mathbb{C}$, $m \neq n$, $|m| \leqslant 1$, $|n| \leqslant 1$, $z, w \in U$ and $g = f^{-1}$ is given by (1.2).

R e m a r k 2.1. It should be remarked that the class $V_\Sigma(\delta, \lambda, m, n; \alpha)$ is a generalization of well-known classes consider earlier. These classes are:

(1) For $\delta = 0$, the class $V_\Sigma(\delta, \lambda, m, n; \alpha) = \angle_\Sigma^\lambda(m, n, \alpha)$, which was introduced by Mazi and Opoola, see [11];
(2) For $\delta = n = 0$ and $m = 1$, the class $V_\Sigma(\delta, \lambda, m, n; \alpha) = \mathfrak{L}B_\Sigma^\lambda(\alpha)$, which was given by Joshi et al. in [8];
(3) For $n = 0$ and $\lambda = m = 1$, the class $V_\Sigma(\delta, \lambda, m, n; \alpha) = M_\Sigma(\alpha, \delta)$, which was investigated by Liu and Wang, see [9];
(4) For $\delta = n = 0$ and $\lambda = m = 1$, the class $V_\Sigma(\delta, \lambda, m, n; \alpha) = S_\Sigma^*(\alpha)$, which was studied by Brannan and Taha, see [3].

We begin by finding the estimates on the coefficients $|a_2|$ and $|a_3|$ for functions in the class $V_\Sigma(\delta, \lambda, m, n; \alpha)$.

Theorem 2.1. *Let* $f \in V_\Sigma(\delta,\lambda,m,n;\alpha)$ $(0<\alpha\leqslant 1,\ 0\leqslant\delta\leqslant 1,\ \lambda\geqslant 1,\ m,n\in\mathbb{C},$ $m\neq n,\ |m|\leqslant 1,\ |n|\leqslant 1)$ *be given by* (1.1). *Then*

$$|a_2| \leqslant \frac{2\alpha}{\sqrt{|2\alpha(\Upsilon(\delta,\lambda,m,n)-mn)+(1-\alpha)(\delta+1)^2(2\lambda-m-n)^2|}}$$

and

$$|a_3| \leqslant \frac{4\alpha^2}{(\delta+1)^2(2\lambda-m-n)^2}+\frac{2\alpha}{(2\delta+1)(3\lambda-m^2-n^2-mn)},$$

where

$$\Upsilon(\delta,\lambda,m,n)=\delta((m^2+n^2+4mn)-6\lambda(m+n-\lambda))+\lambda(1-2(m+n-\lambda)). \tag{2.3}$$

Proof. It follows from conditions (2.1) and (2.2) that

$$(1-\delta)\frac{(m-n)z(f'(z))^\lambda}{f(mz)-f(nz)}+\delta\frac{(m-n)((zf'(z))')^\lambda}{(f(mz)-f(nz))'}=(p(z))^\alpha \tag{2.4}$$

and

$$(1-\delta)\frac{(m-n)w(g'(w))^\lambda}{g(mw)-g(nw)}+\delta\frac{(m-n)((wg'(w))')^\lambda}{(g(mw)-g(nw))'}=(q(w))^\alpha, \tag{2.5}$$

where $g=f^{-1}$ and p, q in $\mathcal{P}$ have the following series representations:

$$p(z)=1+p_1z+p_2z^2+p_3z^3+\ldots \tag{2.6}$$

and

$$q(w)=1+q_1w+q_2w^2+q_3w^3+\ldots \tag{2.7}$$

Comparing the corresponding coefficients of (2.4) and (2.5) yields

$$(\delta+1)(2\lambda-m-n)a_2=\alpha p_1, \tag{2.8}$$

$$(2\delta+1)(3\lambda-m^2-n^2-mn)a_3 + (3\delta+1)((m+n)^2-2\lambda(m+n-\lambda+1))a_2^2=\alpha p_2+\frac{\alpha(\alpha-1)}{2}p_1^2, \tag{2.9}$$

$$-(\delta+1)(2\lambda-m-n)a_2=\alpha q_1 \tag{2.10}$$

and

$$((6\lambda-m^2-n^2)-2\lambda(m+n-\lambda+1)-\delta(6\lambda(m+n-\lambda-1)+(m-n)^2))a_2^2 -(2\delta+1)(3\lambda-m^2-n^2-mn)a_3=\alpha q_2+\frac{\alpha(\alpha-1)}{2}q_1^2. \tag{2.11}$$

In view of (2.8) and (2.10), we conclude that

$$p_1 = -q_1 \tag{2.12}$$

and

$$2(\delta+1)^2(2\lambda-m-n)^2a_2^2 = \alpha^2(p_1^2+q_1^2). \tag{2.13}$$

Also, by using (2.9) and (2.11), together with (2.13), we find that

$$\begin{aligned}2(\delta((m^2+n^2+4mn)-6\lambda(m+n-\lambda))+\lambda(1-2(m+n-\lambda))-mn)a_2^2 \\ = \alpha(p_2+q_2)+\frac{\alpha(\alpha-1)}{2}(p_1^2+q_1^2) \\ = \alpha(p_2+q_2)+\frac{(\alpha-1)(\delta+1)^2(2\lambda-m-n)^2}{\alpha}a_2^2.\end{aligned}$$

Further computations show that

$$a_2^2 = \frac{\alpha^2(p_2+q_2)}{2\alpha(\Upsilon(\delta,\lambda,m,n)-mn)+(1-\alpha)(\delta+1)^2(2\lambda-m-n)^2}, \tag{2.14}$$

where $\Upsilon(\delta,\lambda,m,n)$ is given by (2.3).

By taking the absolute value of (2.14) and applying Lemma 1.1 for the coefficients p_2 and q_2, we have

$$|a_2| \leqslant \frac{2\alpha}{\sqrt{|2\alpha(\Upsilon(\delta,\lambda,m,n)-mn)+(1-\alpha)(\delta+1)^2(2\lambda-m-n)^2|}}.$$

To determine the bound on $|a_3|$, by subtracting (2.11) from (2.9), we get

$$2(2\delta+1)(3\lambda-m^2-n^2-mn)(a_3-a_2^2) = \alpha(p_2-q_2)+\frac{\alpha(\alpha-1)}{2}(p_1^2-q_1^2). \tag{2.15}$$

Now, substituting the value of a_2^2 from (2.13) into (2.15) and using (2.12), we deduce that

$$a_3 = \frac{\alpha^2(p_1^2+q_1^2)}{2(\delta+1)^2(2\lambda-m-n)^2}+\frac{\alpha(p_2-q_2)}{2(2\delta+1)(3\lambda-m^2-n^2-mn)}. \tag{2.16}$$

Taking the absolute value of (2.16) and applying Lemma 1.1 once again for the coefficients p_1, p_2, q_1 and q_2, it follows that

$$|a_3| \leqslant \frac{4\alpha^2}{(\delta+1)^2(2\lambda-m-n)^2}+\frac{2\alpha}{(2\delta+1)(3\lambda-m^2-n^2-mn)}.$$

□

R e m a r k 2.2. In Theorem 2.1, if we choose

(1) $\delta = 0$, then we have the results which were given by Mazi and Opoola in [11], Theorem 1;
(2) $\delta = n = 0$ and $m = 1$, then we have the results obtained by Joshi et al. in [8], Theorem 1;
(3) $n = 0$ and $\lambda = m = 1$, then we obtain the results obtained by Liu and Wang in [9], Theorem 2.2;
(4) $\delta = n = 0$ and $\lambda = m = 1$, then we get the results obtained by Murugusundaramoorthy et al. in [12], Corollary 6.

3. Coefficient estimates for the function class $V_\Sigma^*(\delta, \lambda, m, n; \beta)$

Definition 3.1. A function $f \in \Sigma$ given by (1.1) is said to be in the class $V_\Sigma^*(\delta, \lambda, m, n; \beta)$ if the following conditions are satisfied:

$$\operatorname{Re}\Big\{(1-\delta)\frac{(m-n)z(f'(z))^\lambda}{f(mz)-f(nz)} + \delta\frac{(m-n)((zf'(z))')^\lambda}{(f(mz)-f(nz))'}\Big\} > \beta \tag{3.1}$$

and

$$\operatorname{Re}\Big\{(1-\delta)\frac{(m-n)w(g'(w))^\lambda}{g(mw)-g(nw)} + \delta\frac{(m-n)((wg'(w))')^\lambda}{(g(mw)-g(nw))'}\Big\} > \beta, \tag{3.2}$$

where $0 \leqslant \beta < 1$, $0 \leqslant \delta \leqslant 1$, $\lambda \geqslant 1$, $m \neq n$, $|m| \leqslant 1$, $|n| \leqslant 1$, $z, w \in U$ and $g = f^{-1}$ is given by (1.2).

R e m a r k 3.1. It should be remarked that the class $V_\Sigma^*(\delta, \lambda, m, n; \beta)$ is a generalization of well-known classes consider earlier. These classes are:

(1) For $\delta = 0$, the class $V_\Sigma^*(\delta, \lambda, m, n; \beta) = \angle_\Sigma^\lambda(m, n, \beta)$, which was introduced by Mazi and Opoola, see [11];
(2) For $\delta = n = 0$ and $m = 1$, the class $V_\Sigma^*(\delta, \lambda, m, n; \beta) = \mathfrak{L}B_\Sigma(\lambda, \beta)$, which was given by Joshi et al. in [8];
(3) For $n = 0$ and $\lambda = m = 1$, the class $V_\Sigma^*(\delta, \lambda, m, n; \beta) = B_\Sigma(\beta, \delta)$, which was investigated by Liu and Wang, see [9];
(4) For $\delta = n = 0$ and $\lambda = m = 1$, the class $V_\Sigma^*(\delta, \lambda, m, n; \beta) = S_\Sigma^*(\beta)$, which was studied by Brannan and Taha, see [3].

In this section, we find the estimates on the coefficients $|a_2|$ and $|a_3|$ for functions in the class $V_\Sigma^*(\delta, \lambda, m, n; \beta)$.

Theorem 3.1. *Let* $f \in V_\Sigma^*(\delta,\lambda,m,n;\beta)$ $(0 \leqslant \beta < 1,\ 0 \leqslant \delta \leqslant 1,\ \lambda \geqslant 1,\ m,n \in \mathbb{C},$ $m \neq n,\ |m| \leqslant 1,\ |n| \leqslant 1)$ *be given by* (1.1). *Then*

$$|a_2| \leqslant \frac{\sqrt{2(1-\beta)}}{\sqrt{|\delta((m^2+n^2+4mn)-6\lambda(m+n-\lambda))+\lambda(1-2(m+n-\lambda))-mn|}}$$

and

$$|a_3| \leqslant \frac{4(1-\beta)^2}{(\delta+1)^2(2\lambda-m-n)^2}+\frac{2(1-\beta)}{(2\delta+1)(3\lambda-m^2-n^2-mn)}.$$

P r o o f. In the light of the conditions (3.1) and (3.2), there are $p,q \in \mathcal{P}$ such that

$$(3.3)\qquad (1-\delta)\frac{(m-n)z(f'(z))^\lambda}{f(mz)-f(nz)}+\delta\frac{(m-n)((zf'(z))')^\lambda}{(f(mz)-f(nz))'}=\beta+(1-\beta)p(z)$$

and

$$(3.4)\qquad (1-\delta)\frac{(m-n)w(g'(w))^\lambda}{g(mw)-g(nw)}+\delta\frac{(m-n)((wg'(w))')^\lambda}{(g(mw)-g(nw))'}=\beta+(1-\beta)q(w),$$

where $p(z)$ and $q(w)$ have the forms (2.6) and (2.7), respectively. Comparing the corresponding coefficients in (3.3) and (3.4) yields

$$(3.5)\qquad (\delta+1)(2\lambda-m-n)a_2=(1-\beta)p_1,$$

$$(3.6)\qquad (2\delta+1)(3\lambda-m^2-n^2-mn)a_3 + (3\delta+1)((m+n)^2-2\lambda(m+n-\lambda+1))a_2^2=(1-\beta)p_2,$$

$$(3.7)\qquad -(\delta+1)(2\lambda-m-n)a_2=(1-\beta)q_1$$

and

$$(3.8)\quad ((6\lambda-m^2-n^2)-2\lambda(m+n-\lambda+1)-\delta(6\lambda(m+n-\lambda-1)+(m-n)^2))a_2^2 - (2\delta+1)(3\lambda-m^2-n^2-mn)a_3=(1-\beta)q_2.$$

From (3.5) and (3.7), we get

$$(3.9)\qquad p_1=-q_1$$

and

$$(3.10)\qquad 2(\delta+1)^2(2\lambda-m-n)^2a_2^2=(1-\beta)^2(p_1^2+q_1^2).$$

Adding (3.6) and (3.8), we obtain

$$(3.11)\qquad 2(\delta((m^2+n^2+4mn)-6\lambda(m+n-\lambda)) + \lambda(1-2(m+n-\lambda))-mn)a_2^2=(1-\beta)(p_2+q_2).$$

Hence, we find that

$$a_2^2=\frac{(1-\beta)(p_2+q_2)}{2(\delta((m^2+n^2+4mn)-6\lambda(m+n-\lambda))+\lambda(1-2(m+n-\lambda))-mn)}.$$

By applying Lemma 1.1 for the coefficients p_2 and q_2, we deduce that

$$|a_2|\leqslant\frac{\sqrt{2(1-\beta)}}{\sqrt{|\delta((m^2+n^2+4mn)-6\lambda(m+n-\lambda))+\lambda(1-2(m+n-\lambda))-mn|}}.$$

To determine the bound on $|a_3|$, by subtracting (3.8) from (3.6), we get

$$2(2\delta+1)(3\lambda-m^2-n^2-mn)(a_3-a_2^2)=(1-\beta)(p_2-q_2),$$

or equivalently

$$(3.12)\qquad a_3=a_2^2+\frac{(1-\beta)(p_2-q_2)}{2(2\delta+1)(3\lambda-m^2-n^2-mn)}.$$

Substituting the value of a_2^2 from (3.10) into (3.12), it follows that

$$a_3=\frac{(1-\beta)^2(p_1^2+q_1^2)}{2(\delta+1)^2(2\lambda-m-n)^2}+\frac{(1-\beta)(p_2-q_2)}{2(2\delta+1)(3\lambda-m^2-n^2-mn)}.$$

By applying Lemma 1.1 once again for the coefficients p_1, p_2, q_1 and q_2, we deduce that

$$|a_3|\leqslant\frac{4(1-\beta)^2}{(\delta+1)^2(2\lambda-m-n)^2}+\frac{2(1-\beta)}{(2\delta+1)(3\lambda-m^2-n^2-mn)}.$$

□

Remark 3.2. In Theorem 3.1, if we choose

(1) $\delta=0$, then we have the results which were given by Mazi and Opoola, see [11], Theorem 2;
(2) $\delta=n=0$ and $m=1$, then we have the results obtained by Joshi et al. [8], Theorem 2;
(3) $n=0$ and $\lambda=m=1$, then we obtain the results obtained by Liu and Wang, see [9], Theorem 3.2;
(4) $\delta=n=0$ and $\lambda=m=1$, then we get the results obtained by Murugusundaramoorthy et al. in [12], Corollary 7.

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